Public Allocation and Private Supply: A Housing Assignment Approach

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Abstract

How does public housing allocation affect private rents and supply across quality segments? Motivated by patterns in Hong Kong's rental market, we develop a tractable two-quality housing assignment model with rationed public housing, endogenous construction, and costly conversion of existing dwellings. With imperfect targeting, more generous public housing shifts private demand toward low quality. When new construction is high quality and conversion is sufficiently costly, low-quality private rents rise while high-quality rents fall. In a calibration to Hong Kong, raising public rents or converting public flats to private rental benefits low-quality private renters, but subsidy losses outweigh these gains: expected renter utility falls in every income group when fiscal proceeds are not redistributed.

Keywords: non-market housing allocation, rent control, housing assignment, market rents
JEL: L33, O18, P25, R31, R38

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1 Introduction

Housing affordability is a central concern for policymakers (Glaeser and Gyourko 2018; Hsieh and Moretti 2019; Baum-Snow 2023), and public housing is an important response. Yet many low-income households cannot obtain a public flat and must rent privately. Does greater generosity toward public tenants also improve affordability for those left outside, or can it raise their housing costs? Existing research shows that non-market allocation can influence private-market prices and that housing interventions can have heterogeneous effects across market segments (Fallis and Smith 1984; Wang 2011; Mense, Michelsen and Kholodilin 2023; Monras and Montalvo 2025). Less clear is how these effects differ across housing qualities when private suppliers can change what they build and how existing dwellings are used.

This paper develops a theory of how public housing rationing affects rents across housing quality segments. We show that greater public housing generosity can raise rents for poorer households left in the private market, even as it lowers rents for higher-quality housing. Our assignment model treats dwellings as indivisible and allows their quality composition to respond to developers' construction choices and landlords' decisions to subdivide or combine existing units. We derive a simple condition determining when these supply responses produce opposing rent movements across quality segments and when both rents fall.

Our theory is motivated by patterns in Hong Kong's rental housing market between 2006 and 2016. Roughly a third of the population lived in public housing, where rents remained comparatively rigid as private rents rose sharply. Our analysis of census microdata and rental indices documents three patterns over this period. Public occupancy fell among the poorest households and rose among middle-income households. Rents increased much faster for low-quality private dwellings, and these dwellings accounted for a growing share of the private rental stock. With new construction slow and subdivision offering a way to create smaller rental units, Hong Kong highlights the importance of studying public allocation alongside adjustment in the quality composition of private supply.

Households in the model differ in income and sort into dwellings of two qualities. Public housing provides low-quality units at a subsidized rent, with allocation combining income targeting and a lottery among applicants not served by the targeted pool. Private rents clear

both quality markets. We establish existence and uniqueness of equilibrium under the stated conditions and characterize the effects of changes in public rents and public housing capacity.

Lower public rents attract applicants who would otherwise rent high-quality private housing. With public capacity fixed and targeting imperfect, their admission reduces poorer applicants' chances of obtaining a flat. Private demand therefore shifts toward low quality even though the total number of private renters is unchanged. When new construction is high quality, this shift lowers the construction-input price and high-quality rent while encouraging subdivision. Low-quality rent rises when conversion is sufficiently costly for the demand shift to outweigh the reduction in input costs. With perfect targeting, public flats go to households who would otherwise rent low quality, so this crowding channel disappears.

We calibrate the model to Hong Kong's 2016 rental market to quantify the rent effects and their distribution across households. Census data set the income distribution, housing qualities, public capacity and public rent. We fit preferences and targeting jointly to private rents, housing choices and public occupancy across income groups. Historical construction and rent changes and external subdivision evidence guide the benchmark supply parameters, separately from the fit to rents and occupancy. Under the benchmark supply parameters, conversion is sufficiently costly for the two private rents to move in opposite directions. About one in eight public flats accommodates a household that would otherwise rent high-quality private housing.

We compare the longer-run incidence of two reforms after households and suppliers have adjusted: increasing public rents by 50% and transferring 10% of public flats to private rental. Each transferred flat adds one low-quality private dwelling and reduces public capacity by one; this experiment is distinct from new public construction or sales to sitting tenants. In the benchmark, these reforms lower low-quality private rents by 2.4% and 2.5%, respectively, while raising high-quality rents by 0.5% and 0.6%. Alternative supply calibrations change how the adjustment is divided between the two rents.

These private-market gains come with reduced public assistance. Higher public rents reduce the subsidy per flat, while transferring flats to private rental reduces access to subsidized housing. When fiscal proceeds are not redistributed, these losses outweigh the private-rent gains: mean expected renter utility falls in every income quintile. In the poorest

quintile, it falls by 3.7% under the rent increase and 1.9% under the stock transfer. The calibration thus illustrates a distributional trade-off: reforms that improve affordability for low-income private renters can reduce expected utility for low-income renters as a group.

Our contribution to the literature is fourfold. First, we extend housing assignment models by allowing the quality composition of supply to respond through construction and conversion. Määttänen and Terviö (2014) use a housing assignment model to understand the effects of income inequality on housing prices. Landvoigt, Piazzesi and Schneider (2015) use a similar approach to study the unequal effects of credit expansion across quality segments. Määttänen and Terviö (2022) study the welfare effects of housing transaction taxes using an assignment model with transaction costs. These assignment models take the distribution of housing qualities as given. Epple, Quintero and Sieg (2020) allow housing supply at each quality to respond to its own price, but abstract from direct cross-price supply responses across qualities. Our framework generates these responses through developers' construction choices and landlords' costly subdivision and combination of existing dwellings. Favilukis, Mabille and Van Nieuwerburgh (2023) endogenize housing construction across locations, treating housing within each location as divisible space. Our framework distinguishes discrete housing qualities and models costly conversion of existing dwellings alongside new construction.

Second, we contribute to the theory of non-market housing allocation and private-market spillovers. In an early contribution, Gould and Henry (1967) develop a general equilibrium model showing that price controls can either raise or lower the price of a substitute good. Fallis and Smith (1984) emphasize that the effect of rent control on uncontrolled rents depends on the allocation mechanism in the controlled sector. Wang (2011) shows how the nature of prior state misallocation determines the price effects of housing privatization. In Wang's framework, housing services are homogeneous and divisible, with a single price per unit of services in the private market. Our mechanism instead emphasizes the sorting of households into imperfectly divisible dwellings: households occupy discrete units, and private rents clear separate quality markets. Monras and Montalvo (2025) also model heterogeneous spillovers across housing qualities: a cap binding on high-quality rentals induces landlords to withdraw units and redirects demand toward low-quality housing, raising its rent. In our setting, allocation of a separate public stock shifts demand across two market-clearing private quality

segments, with construction and conversion determining the rent responses.[1]

Third, we complement empirical research on housing spillovers with descriptive evidence and a calibration linking public allocation to rent changes and renter incidence across quality segments. Earlier studies examine average price and rent effects in different policy settings. Wang (2011) finds that privatization of state housing in China increased housing consumption among former state tenants and raised equilibrium prices in the private market. Autor, Palmer and Pathak (2014) find that rent control in Cambridge lowered property values in both controlled and nearby uncontrolled housing, while Mense, Michelsen and Kholodilin (2023) provide quasi-experimental evidence that rent control in Germany raised uncontrolled rents. Monras and Montalvo (2025) document heterogeneous effects of Catalonia's rent cap along the distribution of rents relative to unit-specific reference prices: rents fall for units initially above their reference price but rise for those below it, with rental supply declining primarily among units above the reference price. Related empirical work quantifies the welfare costs of housing misallocation. Glaeser and Luttmer (2003) compare housing consumption across demographic groups in rent-controlled and free-market locations to assess misallocation and its welfare costs beyond those arising from reduced supply. Buurma-Olsen et al. (2025) quantify these costs in Dutch public housing relative to an efficient reassignment of the same stock among the same public tenants, finding substantial differences in housing consumption but modest aggregate welfare losses.

Finally, we contribute to the literature on Hong Kong's housing affordability crisis, which intensified during 2006–2016 amid slow housing construction and rapidly rising private rents. Prior research examines the divergence between housing costs and domestic economic fundamentals (Leung, Ng and Tang 2020*b*), external demand pressures (Taghizadeh-Hesary et al. 2020), and the limited effectiveness of transaction taxes in cooling housing prices (Agarwal et al. Forthcoming), while emphasizing constraints on land supply (Leung, Ng and Tang 2020*a*). We bring public housing allocation into this discussion by linking who receives subsidized flats to the housing costs faced by excluded renters and the composition of private

[1]In related mechanism-design work, Kang (2023) studies public options in a continuous-quality model and shows that equilibrium effects can make rationing optimal. We study specified housing-policy changes under a targeting-plus-lottery rule. Our model also incorporates a durable stock and costly conversion, so competitive rents need not equal the costs of producing new dwellings.

supply.

2 Institutional Context

A large fraction of Hong Kong's population lives in Public Rental Housing (PRH), subsidized government-built units whose rents are highly rigid. PRH units are typically 300-400 square feet in size and are assigned through a first-come, first-served waiting list system. Applicants must satisfy both income and asset tests to receive a PRH unit. Once admitted, public tenants can indefinitely retain their units at heavily subsidized rates.[2] In the broader 2006 census sample, 32.0% of persons lived in PRH.[3] Rents increased discretely with household income: renters paid 1.5 times or double rent if their incomes exceeded double or triple the relevant entry income thresholds. Rents were adjusted every two years according to citywide income growth and capped at a maximum of 10% every two years.[4] By contrast, private-sector rents were unregulated, following the abolition of Hong Kong's rent control laws in 1998.

Our study period of 2006–2016 followed a marked slowdown in housing construction. The 1998 Asian Financial Crisis precipitated a deep recession and plummeting property prices in Hong Kong. In response, the Hong Kong Government sharply repositioned its housing policy away from aggressive urban development in 2002, by pausing land auctions and slowing down public housing construction. New housing construction sharply declined in the early 2000s and remained much lower for the subsequent two decades. The average number of new housing units completed per year between 1997 and 2003 was 70,900, but it declined to an average of 29,300 between 2004 and 2020. Despite the slowdown in construction, Hong Kong's population steadily grew at 0.6 percent per year.[5]

Slow construction and continued population growth coincided with a sharp increase in private-sector rents. Figure 1 shows that public housing rents changed very little between

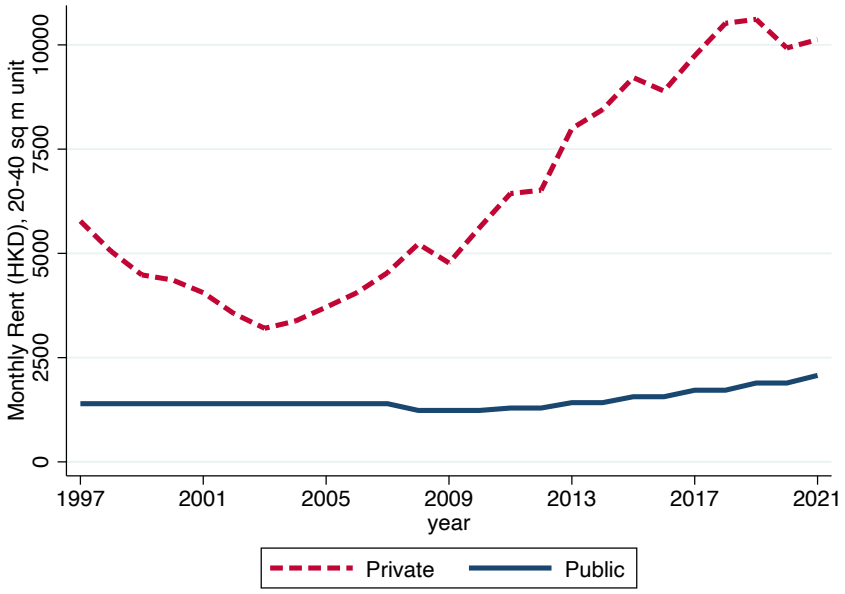
[2]Households are allowed to remain in assigned units unless their incomes exceed the initial income limit by five times.

[3]This share excludes records with tenure marked not applicable, but retains live-in domestic workers, unrelated persons and zero-income households. The 5% sample contains 106,301 public renters among 332,378 persons, giving 31.98%.

[4]Prior to 2007, PRH rents were fixed and were not adjusted at all.

[5]Appendix Figures A1 and A2 report the construction and population-growth series.

Figure 1: Trends in public and private rents



Note: Figure plots the trend of rent in PRH units and private housing. Private housings are constrained to those between 20 and 40 square meters and the private rental indices are weighted by the number of PRH units in each region in 2016 so as to be comparable with the PRH rental indices.

2006 and 2016, rising from $1,396 to only $1,563 per month. However, over the same period, the rents of comparable private-sector units grew from $4,056 to $8,893 per month, reaching a level that is nearly six times that of public-sector units.

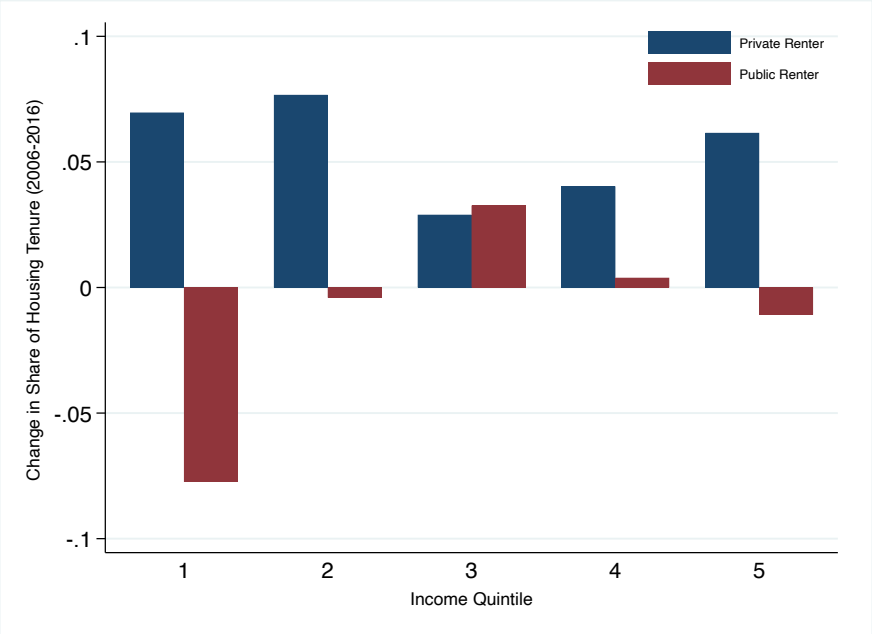
Hong Kong's rental market combines the central institutional features of rationing models: public rents are administratively set below market levels, access to public housing is rationed, and private rents respond to market conditions. This setting motivates our analysis of how public allocation interacts with private demand and supply across housing qualities.

3 Stylized Facts

We document three stylized facts about Hong Kong's housing market between 2006 and 2016: changes in public occupancy across income groups, private rent growth across quality segments, and the composition of private rental housing.[6]

[6]Supporting exhibits are collected in Appendix A.

Figure 2: Change in housing tenure type share by equivalized income quintile



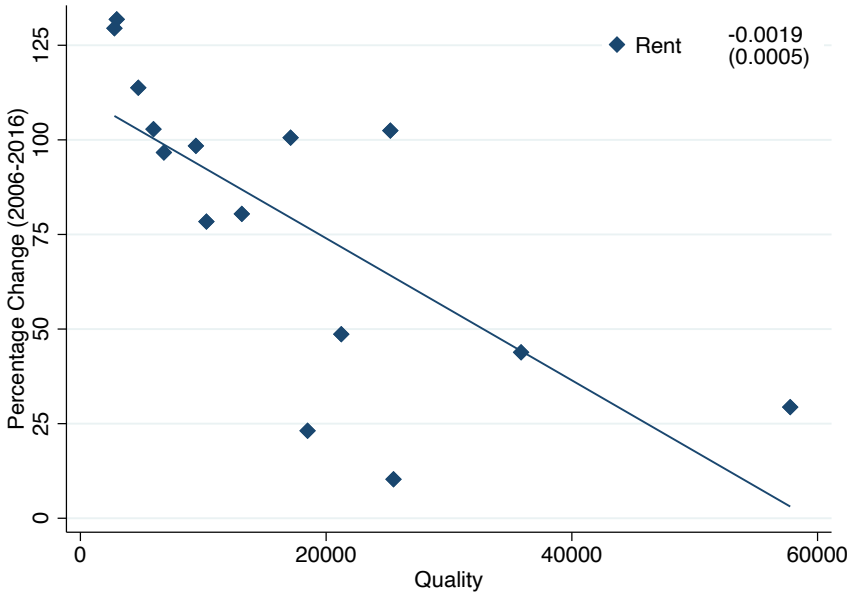
Note: Figure plots the change in housing tenure type share by household equivalized income quintile and housing tenure type. Only households with household head aged between 20 and 60 are included. The housing tenure type share is defined as the ratio of the households living in each type over the total households by income quintile. Households are divided into two housing tenure types: private renters and public renters. The within-group change is calculated as the difference in share between 2006 and 2016 by each household equivalized income quintile.

Fact 1: Public occupancy fell among the poorest households and rose among middle-income households. Figure 2 shows that the public renter share in the lowest income quintile *declined* by over 6.0 percentage points between 2006 and 2016. Meanwhile, the public renter share in the middle income quintile *increased* by around 3 percentage points. Public housing units are predominantly small: roughly four-fifths of PRH units are below 40 m^2, compared with roughly one-third of private units.[7] The changes show that public housing coverage declined at the bottom of the income distribution while expanding in the middle. The widening rent gap increased the financial value of retaining a public unit, providing a potential link between private rents and the composition of public tenants. The model below captures the role of public occupancy across income groups through its targeting parameter.

Fact 2: Low-quality rents disproportionately increased. Alongside the changes in public occupancy, private rent growth was substantially greater at the low-quality end of the

[7]Appendix Table A2 reports the size distributions for 2005–2020.

Figure 3: Change in rents by quality segment



Note: Figure plots percentage change in rents between 2006 and 2016 against unit quality. Rental indices are observed from RVD data. Each dot represents one quality segment, which is comprised of one region and one unit class. Set of regions contains Hong Kong Island, Kowloon and Territories. There are 5 area classes in total, from A to E. The horizontal axis shows the unit quality. Regression coefficients and standard error (in parenthesis) are included in the legend.

market.

To measure rent growth in different quality segments, we use rent indices from the Hong Kong Rating and Valuation Department (RVD), available separately for 15 quality segments.[8] For each segment, we define mean housing quality as the mean rent in 2006 and calculate it as the mean observed rents of households in the 2016 Population Census renting units in the segment, divided by the growth in the segment-specific RVD rent index between 2006 and 2016. We then regress the percentage change in rent and the log of quality (i.e., base year rents) to assess whether rent growth was faster in the low-quality segments.

Figure 3 provides a visualization, showing that rents for the lowest-quality units increased by 129.5% between 2006 and 2016, while rents for the highest-quality units rose by only 29.4%. House prices exhibit a similar growth gradient, including among properties sold in both 2006 and 2016. Growth in price-to-rent ratios, however, shows little systematic variation

[8]The 15 quality segments are formed by crossing three geographic regions (Hong Kong Island, Kowloon, New Territories) with five size categories (from under 40 m^2 to over 160 m^2).

across quality. Thus the stronger price growth at the low-quality end is not accompanied by a clear gradient in price growth relative to rent growth.[9]

Fact 3: Low-quality units accounted for a larger share of private rental housing. Figure 4 shows the distribution of private rental housing quality in 2006 and 2016. As before, quality is defined as the unit's 2006 rent. For 2016, we divide each observed Census rent by the 2006–2016 rent growth in the unit's region and area class, from the RVD rent indices. The 2016 distribution places more mass at the low-quality end than the 2006 distribution and exhibits a distinct low-quality cluster alongside the higher-quality cluster. This shift in composition accompanies the stronger rent growth in low-quality segments documented in Fact 2.

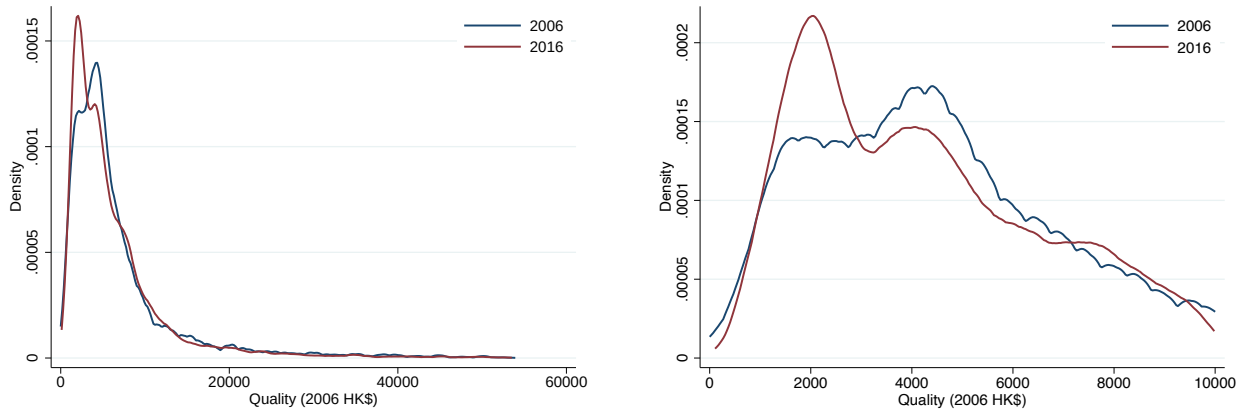
Subdivision offers a supply margin through which the stock of low-quality rental units can expand. Subdivided units created by partitioning a single apartment into multiple self-contained spaces often involve the addition of independent kitchens and bathrooms. They were concentrated in older urban buildings and tended to be much smaller than formal private units. They were also frequently non-compliant and unsafe. The living conditions in subdivided units attracted growing media attention from 2011.[10] According to the 2021 Population Census, around 3% of Hong Kong's population, or roughly 215,700 people, still lived in units smaller than 13.5 m^2, the size of a car parking space. We therefore include subdivision as a margin of supply adjustment, alongside new construction and the combination of existing units.

Theoretical implications. Existing models of rationing spillovers emphasize how below-market rents affect housing allocation across income groups (e.g., Glaeser and Luttmer 2003; Wang 2011). Fact 1 motivates attention to the distribution of public occupancy, while Facts 2 and 3 highlight adjustment across quality segments in the private market. These descriptive patterns do not identify the effect of public allocation on private rents or the supply

[9] Appendix Figure A3 reports price growth in Panel A and price-to-rent growth in Panel B; the latter's fitted slope is small relative to its standard error. Figure A4 compares price growth for repeat-sale properties by their initial transaction value.

[10] Appendix Figures A5 and A6 illustrate subdivision layouts and living conditions using examples from 2017 and 2024; they do not measure prevalence during 2006–2016. Figure A7 reports newspaper and magazine headline mentions, a measure of media attention.

Figure 4: Quality Distribution of Private Rental Housing



(a) Full Distribution

(b) Trimmed Distribution

Notes: Panel (a) plots the full distribution of private rental housing quality. Panel (b) plots the same distribution with the upper tail trimmed at a quality level of HK$10,000 (in 2006 Hong Kong dollars).

responses underlying the changes. They motivate two features of our framework: discrete housing qualities, which allow rents to move differently across segments, and endogenous construction and conversion, which allow the quality composition of supply to respond. The model characterizes these interactions, and the calibration quantifies their implications under specified supply responses.

4 Housing Assignment with Endogenous Supply

This section develops an assignment model of a private housing market with two discrete qualities, common Cobb–Douglas preferences, and endogenous supply. Households differ in total income and equivalized household size. Each household occupies one dwelling. Developers choose which quality to build, and landlords decide whether to transform the existing stock across quality tiers. Household size enters preferences through equivalized consumption and housing quality.

4.1 Setup

Households. There is a measure n of households. A household is characterised by total income I and an exogenous equivalized size $s > 0$. All households share the same Cobb–

Douglas parameter $\alpha \in (0,1)$ and derive utility

$$u(c,q;s) = \left(\frac{c}{s}\right)^{\alpha}\left(\frac{q}{s}\right)^{1-\alpha} = \frac{c^{\alpha}q^{1-\alpha}}{s}, \tag{1}$$

where $c \geq 0$ is total household non-housing consumption and q is the housing quality of the dwelling. Rents are nonnegative. Each household rents one dwelling of type $j \in \{H, L\}$, with quality q_j and rent r_j, subject to

$$c + r_j \leq I. \tag{2}$$

The price of non-housing consumption is normalised to one. Monotonicity in c makes the budget bind, so utility from dwelling j is

$$U_j(I,s) = \left(\frac{I - r_j}{s}\right)^{\alpha}\left(\frac{q_j}{s}\right)^{1-\alpha} = \frac{(I-r_j)^{\alpha}q_j^{1-\alpha}}{s}. \tag{3}$$

The factor $1/s$ cancels when a household compares dwellings, so its housing choice depends on total income. Here $I \geq r_j$.

The pair (I,s) has a joint probability distribution G on $[I_{\min},\infty)\times\mathcal{S}$, where $I_{\min} > 0$ and $\mathcal{S} \subset (0,\infty)$. Income and size may be correlated. Let

$$F(z) \equiv \int_{\{(I,s):\, I\leq z\}} \mathrm{d}G(I,s) \tag{4}$$

be the marginal distribution of total household income, with continuous density $f > 0$ on $(I_{\min},\infty)$ and $F(z) = 0$ for $z \leq I_{\min}$. Shifted log-normal, shifted Weibull and Pareto distributions are examples for this marginal. Conditions (Stock′)–(Sort′) in Appendix B.2 guarantee $0 < r_L < I_{\min}$, $r_L < r_H$ and $I_{HL} > I_{\min}$ at the baseline equilibrium.

Housing quality. There are two discrete quality levels $q_H = H$ and $q_L = L$, with $H > L > 0$. Quality is measured per dwelling; equivalized quality is q_j/s. Equilibrium rents (r_L, r_H) are determined by market clearing.

Developers. The initial stock of market houses is (h_L^0, h_H^0). New market houses (denoted by the superscript n) are built by a continuum of atomistic, identical developers indexed by

$i \in [0,1]$. Building a unit of quality j requires θ_j units of land and incurs a non-land cost $k_j = \kappa_j \theta_j$, where $\kappa_j \geq 0$ is the non-land cost per unit of land and $\theta_H > \theta_L > 0$. Aggregate input demand is $Q = \theta_H h_H^n + \theta_L h_L^n$, where $h_j^n = \int_0^1 h_{ij}^n \, di$. The input price is determined by market clearing, with a linear inverse supply function $p(Q) = bQ$, $b > 0$.

Each developer is negligible relative to the input market and therefore takes both rents and the input price p as given, solving

$$\max_{h_{iH}^n, h_{iL}^n \geq 0} \big(r_H - k_H - p\,\theta_H\big) h_{iH}^n + \big(r_L - k_L - p\,\theta_L\big) h_{iL}^n. \tag{5}$$

Free entry requires

$$r_j - k_j - p\,\theta_j \leq 0 \quad \text{for } j \in \{H, L\}, \qquad \text{with equality whenever } h_j^n > 0, \tag{6}$$

Aggregate construction is determined by input-market clearing. Define the net return per unit of land in segment j as

$$\rho_j \equiv \frac{r_j}{\theta_j} - \kappa_j, \tag{7}$$

which is the object a developer compares across quality tiers.

Landlord. A representative landlord manages the existing stock by transforming units between quality tiers. Let $\Delta \equiv h_L^l - h_L^0$ denote the net creation of L-quality units through conversion, which may be positive or negative. Each L-quality unit converts into θ_L/θ_H H-quality units, and each H-quality unit converts into θ_H/θ_L L-quality units, conserving land. The landlord incurs a quadratic adjustment cost $\Phi(\Delta) = \frac{\gamma}{2}\Delta^2$ with $\gamma > 0$, and solves

$$\max_{-h_L^0 \leq \Delta \leq (\theta_H/\theta_L) h_H^0} \; r_L\big(h_L^0 + \Delta\big) + r_H\left(h_H^0 - \frac{\theta_L}{\theta_H}\Delta\right) - \frac{\gamma}{2}\Delta^2. \tag{8}$$

4.2 Supply Functions

Landlord supply. When conversion is strictly interior, the first-order condition gives $\Delta^* = \big(r_L - (\theta_L/\theta_H)r_H\big)/\gamma$, and supply is

$$h_L^l(r_L, r_H) = h_L^0 + \frac{1}{\gamma}\left(r_L - \frac{\theta_L}{\theta_H} r_H\right), \tag{9}$$

$$h_H^l(r_L, r_H) = h_H^0 - \frac{\theta_L}{\theta_H \gamma}\left(r_L - \frac{\theta_L}{\theta_H} r_H\right). \tag{10}$$

Developer supply. Combining the zero-profit conditions (6) with input-market clearing $p = bQ$, any quality actually produced satisfies

$$p\,\theta_j = r_j - k_j \implies bQ = \frac{r_j}{\theta_j} - \kappa_j = \rho_j,$$

so the equilibrium input price equals the net return per unit of land in the active segment. Since the input is common across qualities, entry concentrates on whichever quality yields the higher net return per unit of land ρ_j defined in (7); the segment with the lower net return earns a strictly negative margin at that p and attracts no construction. Let $\Theta \equiv \theta_H/\theta_L$ denote the relative input intensity of high-quality housing. Input-market clearing gives $p = \max\{0, \rho_L, \rho_H\}$ and $Q = p/b$. If both returns are nonpositive, there is no construction. When the maximum return is positive, three cases arise.

1. *High-quality production* ($\rho_H > \rho_L$, $\rho_H > 0$): $h_H^n = \frac{r_H - k_H}{b\theta_H^2}$, $\qquad h_L^n = 0$.

2. *Low-quality production* ($\rho_H < \rho_L$, $\rho_L > 0$): $h_H^n = 0$, $\qquad h_L^n = \frac{r_L - k_L}{b\theta_L^2}$.

3. *Co-production* ($\rho_H = \rho_L = \rho^* \geq 0$): both segments break even simultaneously, so the composition of entry is not pinned down at the firm level; aggregate supply is the correspondence

$$S^n(\rho^*) = \left\{(h_L^n, h_H^n) \in \mathbb{R}_+^2 \;\middle|\; \theta_L h_L^n + \theta_H h_H^n = \frac{\rho^*}{b}\right\}, \tag{11}$$

   where $\rho^* = \rho_H = \rho_L$.

The set of rent pairs at which developers are indifferent is the co-production locus:

$$\mathcal{B} \equiv \big\{(r_L, r_H) : \rho_H = \rho_L\big\} = \big\{(r_L, r_H) : r_H = \Theta r_L + \theta_H(\kappa_H - \kappa_L)\big\}, \tag{12}$$

an affine line of slope Θ that passes through the origin if and only if $\kappa_L = \kappa_H$.

Evaluating the landlord's first-order condition on $\mathcal{B}$ and using (12) to substitute for r_H,

$$\Delta^*\Big|_{\mathcal{B}} = \frac{1}{\gamma}\left(r_L - \frac{\theta_L}{\theta_H}\big[\Theta r_L + \theta_H(\kappa_H - \kappa_L)\big]\right) = \frac{\theta_L}{\gamma}\big(\kappa_L - \kappa_H\big) \;\equiv\; \Delta^\dagger, \tag{13}$$

a constant along $\mathcal{B}$, and non-zero whenever $\kappa_L \neq \kappa_H$. The sign follows the cost asymmetry: $\kappa_H > \kappa_L$ gives $\Delta^\dagger < 0$, so the landlord upgrades existing stock rather than relying on costly new high-quality construction. Positive construction of both qualities is thus compatible with non-zero net conversion, whose magnitude γ pins down.

Combining (9)–(10) with the developer supply, total supply for $j \in \{H, L\}$ is $S_j(r_L, r_H) = h_j^l(r_L, r_H) + h_j^n(r_L, r_H)$. In the two strict production regimes, with interior conversion, S_j is single-valued and continuously differentiable; on the affine co-production locus $\mathcal{B}$ it becomes the compact convex correspondence (11) added to the landlord supply.

4.3 Demand Functions

Using (3), a household of any size s is indifferent between H- and L-type housing at income I_{HL} when

$$\frac{(I_{HL} - r_H)^\alpha H^{1-\alpha}}{s} = \frac{(I_{HL} - r_L)^\alpha L^{1-\alpha}}{s}. \tag{14}$$

Cancelling $1/s$ and defining $\delta \equiv (L/H)^{(1-\alpha)/\alpha} \in (0, 1)$, equation (14) yields the closed-form threshold

$$I_{HL}(r_L, r_H) = \frac{r_H - \delta r_L}{1 - \delta}. \tag{15}$$

For $0 \leq r_L < I_{\min}$ and $r_H > r_L$, the cutoff satisfies $I_{HL} > r_H$. For every s, households with $I > I_{HL}$ rent H-type housing and those with $I < I_{HL}$ rent L-type; households at the threshold are indifferent and have zero aggregate mass. Integrating over the joint distribution

gives

$$n \int_{\{(I,s):\, I \le I_{HL}\}} \mathrm{d}G(I,s) = nF(I_{HL}).$$

Aggregate demand is therefore

$$D_H(r_L, r_H) = n\big[1 - F(I_{HL}(r_L, r_H))\big], \tag{16}$$

$$D_L(r_L, r_H) = n\, F(I_{HL}(r_L, r_H)). \tag{17}$$

Both functions are continuously differentiable wherever $I_{HL} > I_{\min}$, since f is continuous and positive on the interior of the income support. Below that support, $D_L = 0$ and $D_H = n$.

4.4 Equilibrium

A competitive market equilibrium consists of rents (r_L^*, r_H^*) and allocations such that households maximise utility (1) subject to budget constraint (2), developers and the landlord maximise profit given (5) and (8), the zero-profit conditions (6) hold, and all markets clear:

$$S_L(r_L, r_H) = D_L(r_L, r_H), \tag{18}$$

$$S_L(r_L, r_H) + S_H(r_L, r_H) = n. \tag{19}$$

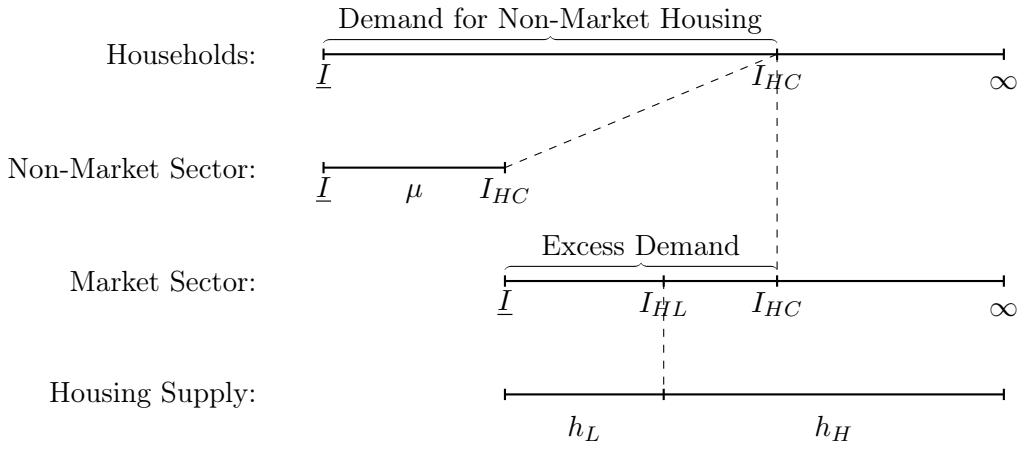
Proposition 1 (Existence and Uniqueness)**.** *Under Conditions* (Stock′)–(Sort′)*, a competitive equilibrium exists and is unique in rents, conversion, and aggregate construction.*

Proof. See Appendix B.2. □

5 A New Theory of Rationing Spillovers

This section adds a non-market housing sector to the model. Unlike prior rent control models (e.g. Wang 2011), housing quality here is discrete rather than continuously variable. We allow for imperfect targeting of subsidised units and characterise how changes in the non-market rent and the measure of non-market housing affect rents in the two market quality segments.

Figure 5: Equilibrium rationing and sorting



5.1 Setup

There is a measure $n > 1$ of households, while the initial housing stock has total mass one. The timing of the model is as follows.

First, a measure μ of non-market housing is fixed, with $\mu = 1 - (h_L^0 + h_H^0)$, so that the market and non-market stocks together exhaust the unit initial stock. All non-market units are of L-quality, with an exogenously set rent $r_C < r_L$. For the local comparative statics we maintain active construction, nonnegative returns and interior conversion. A measure μ is housed in the non-market sector and $n - \mu$ in the market sector.

Second, following Glaeser and Luttmer (2003), allocation of non-market units combines income targeting with random rationing, parameterised by $\lambda \in [0, 1]$. Define the income-targeted group $\mathcal{E} = \{I \mid I_{\min} \leq I \leq I_\mu\}$ as the μ lowest-income households, where I_μ solves $nF(I_\mu) = \mu$, i.e. $F(I_\mu) = \mu/n$. Targeting ranks households by total income I. A targeted pool of $\lambda\mu$ units is reserved for $\mathcal{E}$; the remaining $(1-\lambda)\mu$ units are allocated by lottery among the eligible households *not* served by the targeted pool, so the two groups of recipients are disjoint. Within $\mathcal{E}$, the targeted recipients are selected uniformly. Under excess applications, exactly μ distinct households obtain a non-market unit. The cases $\lambda = 1$ and $\lambda = 0$ recover perfect targeting and pure lottery, respectively.

Third, market rents and the input price are determined in competitive equilibrium.

5.2 Equilibrium

Eligibility threshold. By single-crossing, a household applies for non-market housing if and only if its income falls below the threshold I_{HC} at which it is indifferent between a non-market L-quality unit at rent r_C and a market H-quality unit at rent r_H:

$$\frac{(I_{HC} - r_H)^\alpha H^{1-\alpha}}{s} = \frac{(I_{HC} - r_C)^\alpha L^{1-\alpha}}{s}.$$

Cancelling $1/s$ and solving for income gives

$$I_{HC}(r_C, r_H) = \frac{r_H - \delta r_C}{1 - \delta}. \tag{20}$$

Since $r_C < r_L$, equations (20) and (15) give $I_{HC} > I_{HL}$. We maintain the following local domain:

$$\begin{gathered} 0 < \mu < n, \quad 0 \le r_C < r_L < I_{\min}, \quad r_L < r_H, \quad nF(I_{HL}) > \mu, \\ \rho_L, \rho_H \ge 0, \quad -h_L^0 < \Delta^* < (\theta_H/\theta_L)h_H^0. \end{gathered} \tag{21}$$

with active construction. For the co-production derivatives in Propositions 2 and 3, we additionally require

$$0 < \phi^* \equiv h_L^{n*} < \frac{\rho^*}{b\theta_L}, \qquad \rho_H = \rho_L = \rho^* > 0. \tag{22}$$

Both qualities then have positive construction. Since $nF(I_{HL}) > \mu$, every targeted recipient would otherwise rent market L. Lottery applicants with $I_{HL} < I < I_{HC}$ would otherwise rent market H. Appendix B.3 gives sufficient conditions for existence and uniqueness for every λ under this allocation rule.

Acceptance rate. The mass of eligible applicants is $nF(I_{HC})$; removing the $\lambda\mu$ targeted recipients leaves a lottery pool of mass $nF(I_{HC}) - \lambda\mu$. By (21), $nF(I_{HC}) > nF(I_{HL}) > \mu$. For $\lambda < 1$ the lottery pool therefore exceeds the units on offer. The lottery acceptance rate, which equals zero at $\lambda = 1$, is

$$P_{\text{rand}} = \frac{(1-\lambda)\mu}{nF(I_{HC}) - \lambda\mu} < 1. \tag{23}$$

Market clearing. Competitive sorting in the market sector yields the threshold I_{HL} in (15). Because $I_\mu < I_{HL}$ by the maintained domain (21), the income-targeted group $\mathcal{E}$ is a subset of the market L-quality applicants: $\mathcal{E} \subseteq \{I \le I_{HL}\}$. Households below I_{HL} number $nF(I_{HL})$; of these the $\lambda\mu$ targeted recipients leave the market, and the remaining $nF(I_{HL}) - \lambda\mu$ belong to the lottery pool and lose a fraction P_{rand} to lottery wins. Market demand for L-quality housing is therefore

$$D_L^M(r_L, r_H; r_C, \mu, \lambda) = \big(nF(I_{HL}) - \lambda\mu\big)\left(1 - \frac{(1-\lambda)\mu}{nF(I_{HC}) - \lambda\mu}\right). \tag{24}$$

Market demand for H-quality housing removes random-pool winners who would otherwise have rented high-quality:

$$D_H^M(r_L, r_H; r_C, \mu, \lambda) = n\big(1 - F(I_{HL})\big) - n\big(F(I_{HC}) - F(I_{HL})\big)P_{\text{rand}}. \tag{25}$$

It is straightforward to verify from (24)–(25) that $D_L^M + D_H^M = n - \mu$. The equilibrium (r_L, r_H) satisfies

$$S_L(r_L, r_H) = \big(nF(I_{HL}) - \lambda\mu\big)\left(1 - \frac{(1-\lambda)\mu}{nF(I_{HC}) - \lambda\mu}\right), \tag{26}$$

$$S_L(r_L, r_H) + S_H(r_L, r_H) = n - \mu. \tag{27}$$

5.3 Effects of a Change in the Non-Market Rent

We characterize how a reduction in r_C affects market rents. The developers' production choice depends on the sign of $\rho_H - \rho_L$, so the analysis separates into three regimes.

Suppose r_C decreases while r_L and r_H are held fixed. Some higher-income households who previously avoided non-market housing due to its lower quality may now find the reduced rent attractive enough to apply. The marginal applicant therefore shifts toward households who would otherwise have preferred high-quality market housing, raising the eligibility threshold I_{HC} in (20). Consequently, demand for high-quality housing in the market sector declines. Since non-market units are rationed through random allocation, the increase in applicants lowers the acceptance rate P_{rand} in (23) for all participants. This crowding externality falls

disproportionately on applicants who, if rejected, would have rented market low-quality housing. Even holding I_{HL} fixed, more rejected applicants therefore spill over into the market low-quality segment, creating excess demand for those units. With $n - \mu$ fixed, the shift from market H to market L reduces land requirements because $\theta_L < \theta_H$. In the high-quality production regime, $r_H = k_H + \theta_H p$ falls with the land price, while increased conversion raises the wedge $r_L - (\theta_L/\theta_H) r_H = \gamma\Delta$. Low-quality rent rises precisely when (28) holds: conversion is sufficiently costly for the increase in this wedge to outweigh the fall in land cost. Otherwise r_L falls weakly, remaining unchanged at $\gamma = b\theta_L(\theta_H - \theta_L)$. The threshold depends on supply parameters because aggregate market demand is fixed. In the low-quality production regime, $r_L = k_L + \theta_L p$ falls with the land price; both market rents then decline.

Proposition 2. *Consider a sufficiently small decrease in r_C at an equilibrium satisfying (21), holding μ and G fixed. For $0 \leq \lambda < 1$, the following local derivatives apply within the indicated production regime. Part (iii) also imposes (22).*

(i) High-quality production regime ($\rho_H > \rho_L$): $dr_H/dr_C > 0$. Low-quality rent rises as r_C falls, $dr_L/dr_C < 0$, if and only if

$$\gamma > b\theta_L(\theta_H - \theta_L). \tag{28}$$

(ii) Low-quality production regime ($\rho_H < \rho_L$): $dr_L/dr_C > 0$ and $dr_H/dr_C > 0$, so a decrease in r_C lowers both market rents.

(iii) Interior co-production regime ((22)): $dr_L/dr_C > 0$ and $dr_H/dr_C > 0$, so a decrease in r_C lowers both market rents.

At $\lambda = 1$ (perfect targeting), $dr_L/dr_C = dr_H/dr_C = 0$.

Proof. See Appendix B.4. □

5.4 Effects of a Change in the Measure of Non-Market Housing

We now consider how an increase in the measure of non-market housing μ affects market rents. As μ increases, we hold h_H^0 fixed and set $\partial h_L^0/\partial\mu = -1$, so that the identity $\mu = 1 - (h_L^0 + h_H^0)$

is maintained. Each additional non-market unit withdraws one unit from the low-quality market stock, reducing market supply at fixed rents: $\partial S_L/\partial\mu = -1$ and $\partial S_H/\partial\mu = 0$.

At the same time, a larger non-market measure increases the targeted pool $\lambda\mu$ and the random pool $(1-\lambda)\mu$ available to low-income households, raising the probability that any given eligible applicant secures a non-market unit. This reduces demand for market housing from both segments. In the L-quality market, the direct stock reduction raises excess demand, while improved non-market coverage reduces demand. Their net direct effect is positive when $\lambda < 1$. The adjustment lowers the land price and hence $r_L = k_L + \theta_L p$ in the low-quality production regime; both market rents decline. In the high-quality production regime, r_H falls, while r_L rises if and only if condition (28) holds. At perfect targeting, the direct withdrawal of low-quality market demand exactly offsets the stock reduction.

Proposition 3. *Consider a sufficiently small increase in μ at an equilibrium satisfying (21), holding r_C, h_H^0 and G fixed and setting $dh_L^0/d\mu = -1$. For $0 \le \lambda < 1$, the following local derivatives apply within the indicated production regime. Part (iii) also imposes (22).*

(i) High-quality production regime ($\rho_H > \rho_L$): $dr_H/d\mu < 0$. Low-quality rent rises as μ increases, $dr_L/d\mu > 0$, if and only if condition (28) holds.

(ii) Low-quality production regime ($\rho_H < \rho_L$): $dr_L/d\mu < 0$ and $dr_H/d\mu < 0$, so an increase in μ lowers both market rents.

(iii) Interior co-production regime ((22)): $dr_L/d\mu < 0$ and $dr_H/d\mu < 0$, so an increase in μ lowers both market rents.

At $\lambda = 1$ (perfect targeting), $dr_L/d\mu = dr_H/d\mu = 0$.

Proof. See Appendix B.5. □

6 A Calibration Exercise

We calibrate the model to Hong Kong's 2016 rental market to quantify the mechanism developed in Sections 4 and 5. We match baseline rents and occupancy, assign benchmark supply responses, and compare the longer-run incidence of two reforms after households and suppliers have adjusted.

6.1 The Benchmark Calibration

The sample contains 36,639 tenant households from the 2016 Population By-census, with household heads aged 20 to 60 and income above HK$5,578.78. All monetary values are monthly, in 2006 Hong Kong dollars. Private dwellings are classified into two qualities using rent at 2006 prices and market conditions, with the 2006 median rent as the cutoff. Public flats are assigned the same representative quality as low-quality private dwellings. Census data set the income distribution, representative qualities and floor areas, public capacity and public rent. Public housing accommodates 53.07% of sample households, at a median rent of HK$1,298.54; among private tenants, 47.97% occupy high-quality dwellings.

We jointly choose the preference parameter δ, targeting share λ and baseline low-quality rent r_L to match eight moments: public occupancy among the poorest μ households and in income quintiles Q1–Q4, the private high-quality share and the two private rents. The allocation rule combines a pool reserved for poorer applicants with a lottery, as in Section 5, ranking applicants by equivalized income. Private housing choices depend on total income. The private-quality share helps discipline willingness to pay for quality, while the public-occupancy gradient helps discipline targeting. The two are fitted jointly because preferences also determine who applies for public housing. A higher δ means less willingness to pay for quality, and a higher λ reserves more flats for poorer applicants. The allocation rule summarizes the occupied stock; its response to policy follows from the maintained preferences and rationing mechanism.

Table 1 reports the calibration. The fitted values are $\delta = 0.718$ and $\lambda = 0.17$, with a root mean squared relative moment error of 2.92%. The model overpredicts public occupancy in the highest-income quintile, at 14.40% against 8.72% in the data. It implies that 12.4% of public flats accommodate households that would otherwise rent high-quality private housing. These households connect public allocation to demand across private quality segments. At the fitted rents, households prefer private high quality above total income $I_{HL} = 29{,}417.23$, but prefer a public flat to private high quality below $I_{HC} = 39{,}631.21$. Public recipients between these cutoffs remove demand from the high-quality private market. In the model, 81.5% of households apply for capacity equal to 53.1% of households. Public housing is

Table 1: Parameters and how each is set

Parameter	Value	Set by	Source
Panel A. Demand			
δ, value of quality	0.718	Fitted	Private-H share
r_L, low-quality rent	5,311.71	Fitted	L rent in the census
r_H, high-quality rent	12,111.36	Implied	Zero profit on new H dwellings
Panel B. Public housing			
μ, public flats	0.531	Data	Public share of sample households
r_C, public rent	1,298.54	Data	Median public rent
λ, reserved share	0.17	Fitted	Fall in occupancy from the poorest households to Q4
P_{rand}, lottery rate	0.608	Implied	Fills the μ flats
Panel C. Supply			
b, slope of land supply	9.21	Calibrated	Historical land-rent change relative to construction, assuming a stable schedule
γ, conversion cost	50,095	Calibrated	Mapping from Leung, Choy and Chau (2022), conditional on coefficient scaling and transferability
θ_L, θ_H, floor area (m^2)	29.17; 64.51	Data	Mean floor area of each quality, 2016 census
κ, construction cost per m^2	75.04	Calibrated	Cost handbook, 2% real rate

Notes: Monetary values are monthly 2006 HK dollars. The fitted objects (δ, r_L, λ) jointly match eight 2016 moments. Supply parameters b and γ are benchmark assumptions motivated by historical and external evidence; baseline fit does not identify their values. Implied values follow from the others. Appendix Table A4 lists the inputs behind the supply block.

therefore rationed, with a lottery acceptance rate of 60.8%.

Supply responses are assigned separately from the fit to rents and occupancy. We set $b = 9.21$ by relating the historical change in inferred land rent to a proxy for cumulative rental construction, assuming a stable input-supply schedule. We set $\gamma = 50{,}095$ by mapping the subdivision relationship in Leung, Choy and Chau (2022) into the model's conversion response. The conversion calibration transfers the older-building relationship to the modeled stock and applies the symmetric adjustment technology to both subdivision and combination. The baseline moments do not separately identify these supply responses.[11]

[11] Appendices C.2–C.5 document the supply calibration and alternative stock-accounting scenarios. The subdivision source does not specify the coefficient's scale; the benchmark interprets it as applying to one standard deviation of the rent gap.

High-quality rent follows from zero profit at the calibrated costs. Market clearing and landlord optimality recover the reference stocks, rather than using historical stocks as additional fitting targets. The fitted rents are $r_L = 5{,}311.71$ and $r_H = 12{,}111.36$. Building a low-quality dwelling would cost 5,477.14 per month, so only high-quality dwellings are constructed.[12]

6.2 Supply and the Direction of Spillovers

The two supply margins determine how the change in private demand affects rents. Consider a shift toward high quality with the number of private renters fixed. A higher high-quality rent encourages new construction, adding dwellings, and combination of low-quality dwellings, reducing their number. When construction responds more strongly than combination, low-quality rent must fall to keep the total stock consistent with demand. This is the opposite-sign case in Section 5.

Within the high-quality construction regime, the relative supply response governs how the adjustment is divided between rents:

$$r_L^1 - r_L^0 = \frac{\theta_L}{\theta_H}(1-\chi)(r_H^1 - r_H^0), \qquad \chi \equiv \frac{\gamma}{b\theta_L(\theta_H - \theta_L)}. \tag{29}$$

This restates the theoretical sign condition: rents move in opposite directions when $\chi > 1$, low-quality rent is unchanged at $\chi = 1$, and both rents move in the same direction when $\chi < 1$. The benchmark gives $\chi \simeq 5.3$, so low-quality rent moves approximately HK\$1.93 in the opposite direction for each dollar of change in high-quality rent. The benchmark therefore assigns most of the rent adjustment to the low-quality segment; preferences and allocation determine the size of the demand shift. Equation (29) applies to the public-rent experiment and the specified transfer of public flats into private rental while construction remains high quality. Other production regimes follow the corresponding comparative statics in Section 5.

[12] Appendix C.1 describes the sample and measurement; Appendices C.6–C.9 give the calibration procedure and equilibrium checks. Table A7 reports the full moment fit.

6.3 Two Policy Experiments

We increase public rents by 50% or transfer 10% of public flats to the low-quality private rental stock. Each experiment holds the household population and income distribution, preferences, qualities, costs, supply slopes and targeting parameter fixed. Reference stocks remain fixed except for the public-to-private transfer, while allocation, construction, landlord conversion and rents adjust. Both experiments remain in the high-quality construction regime. Conversion here means a transfer to market rental, rather than a sale to sitting tenants or a change in new public construction.

The experiments compare equilibria after households have had time to move and suppliers have adjusted construction and conversion. This longer-run interpretation matters in Hong Kong, where the average applicant waited 4.7 years at the end of 2016 and about 12,000 flats were recovered from departing tenants in the year to March 2017.[13] The lottery summarizes the resulting allocation of public flats, rather than an immediate reassignment of incumbent tenants.

Rents and allocation. Higher public rents induce some applicants who would otherwise rent high quality to withdraw, improving poorer applicants' access. Private demand shifts toward high quality, lowering low-quality rent by 2.4% and raising high-quality rent by 0.5%. Conversion instead adds more low-quality private supply than demand, because some households losing public access choose high quality. Low-quality rent falls by 2.5% and high-quality rent rises by 0.6% (Table 2).

The low-quality rent reductions are HK$128 and HK$130 per month, respectively. For scale, the observed increase between the censuses was HK$2,324. The experiments therefore generate modest private-rent changes relative to the historical increase, while isolating the allocation channel.

Renter incidence. We measure expected utility before public allocation, averaging utility per equivalent adult with person weights and grouping persons by equivalized household income. A household's expected utility reflects both its probability of receiving a public flat

[13] See Legislative Council (2017); Transport and Housing Bureau (2018). The model abstracts from the transition path and incumbent tenure rights. Its longer-run comparison remains conditional on the inherited stock and conversion technology, rather than describing a dynamic steady state.

Table 2: Incidence of the two benchmark reforms

Change (%)	Public rent +50%	Public stock −10%
Low-quality private rent	−2.4	−2.5
High-quality private rent	0.5	0.6
Expected renter utility, poorest quintile	−3.7	−1.9
Expected renter utility, all renters	−1.0	−0.6

Notes: Percentage changes from the calibrated baseline. Public-stock reductions transfer flats to private low-quality rental. Utility is person-weighted and measured before allocation; fiscal proceeds are not redistributed.

and its private alternative. Changes in group means combine three channels: access to public housing, public rents and private rents.

Higher public rents reduce the subsidy received by tenants; conversion reduces access to subsidized flats. These losses outweigh the private-rent gains in the benchmark: mean expected renter utility falls in every quintile, including by 3.7% and 1.9%, respectively, in the poorest quintile. For the public-rent increase, the decomposition holds tenure fixed to isolate rent effects: utility among the poorest quintile's private low-quality tenants rises by 2.3%, while utility among its public tenants falls by 5.9%. Improved public access contributes only 0.3 percentage points to the poorest quintile's utility change. Under conversion, public rents are unchanged, but reduced access contributes −2.5 percentage points, partly offset by the private-rent gains.[14]

These calculations measure renter incidence with fiscal proceeds left undistributed. The higher-rent experiment raises government receipts by HK$345 per month per sample household. Returning that revenue would introduce a further distributional channel. The calculations exclude landlords, developers and the government, and the reforms are not matched on their fiscal effects, so the comparison does not rank their social welfare.

The magnitudes depend on the supply calibration. With $b = 33.03$, the through-origin alternative, the public-rent increase lowers low-quality rent by only 0.6% and raises high-quality rent by 1.2%. The alternative calibrations reported here preserve the baseline signs. Equation (29) identifies the threshold at which the low-quality response reverses. The

[14]Appendix C.11 defines the utility measure, and Table A8 reports its decomposition. Figure A8 and Tables A9–A10 give the policy paths and associated market adjustments.

quantitative lesson is that the allocation rule determines the demand shift, while relative supply responsiveness determines how that shift is distributed across private rents. In the benchmark, reforms that improve affordability for excluded renters also reduce assistance to public tenants.[15]

7 Conclusion

This paper develops a tractable housing assignment model linking public allocation to endogenous construction and costly conversion of existing dwellings. Allocation determines which households leave private demand, while construction and conversion determine how the resulting demand shift affects rents across quality segments.

With imperfect targeting, more generous public housing can attract households who would otherwise rent high-quality private dwellings and displace poorer applicants into the low-quality private market. When new construction is high quality and conversion is sufficiently costly, low-quality private rents rise while high-quality rents fall. The sign condition makes the role of relative supply responsiveness explicit.

A calibration to Hong Kong's 2016 rental market illustrates the magnitude and distribution of this mechanism. Under the benchmark supply assumptions, raising public rents or transferring public flats to private rental lowers low-quality rents and raises high-quality rents after households and suppliers have adjusted. Low-income private renters gain, but subsidy losses outweigh those gains: expected renter utility falls in every income group when fiscal proceeds are not redistributed. The reforms reduce assistance through higher public rents or reduced access, exposing the trade-off between assisting public tenants and improving affordability for excluded renters.

The policy implication is that the incidence of public housing depends jointly on who receives it and how private supply adjusts across qualities. Assessing a reform therefore requires tracing its effects on public access, private rents and the use of fiscal proceeds. The model provides a framework for these comparisons and shows why aggregate rent effects alone can conceal important differences among low-income households.

[15] Appendix Tables A11–A13 report the alternative calibrations and their policy outcomes.

References

Agarwal, Sumit, Kwong Wing Chau, Maggie Hu and Wayne Xinwei Wan. Forthcoming. "Tobin Tax Policy, Housing Speculation, and Property Market Dynamics." *Management Science* .

Arcadis. 2018. *Construction Cost Handbook: China and Hong Kong*. Hong Kong: Arcadis. Hong Kong cost tables, pp. 9–10; general scope notes, p. 40; tender-price index, p. 49. https://www.arcadis.com/contentassets/934a2cbf81254a22b8d893e40f92c781/construction-cost-handbook-china-and-hong-kong-2018.pdf.

Architectural Services Department. 2024. "Tender Price Index for Building Works and Building Services Tender Price Index." Hong Kong Government. Quarterly indices. https://www.archsd.gov.hk/en/psi/tpi-bw-en.json; https://www.archsd.gov.hk/en/reports/bstpi-published-in-earlier-years.html.

Autor, David H., Christopher J. Palmer and Parag A. Pathak. 2014. "Housing Market Spillovers: Evidence from the End of Rent Control in Cambridge, Massachusetts." *Journal of Political Economy* 122(3):661–717.

Baum-Snow, Nathaniel. 2023. "Constraints on city and neighborhood growth: the central role of housing supply." *Journal of Economic Perspectives* 37(2):53–74.

Buurma-Olsen, Jennifer, Hans R. A. Koster, Jos van Ommeren and Jort Sinninghe Damsté. 2025. "Quantifying Misallocation of Public Housing." *Journal of Public Economics* 242:105272.

Census and Statistics Department. 2021. "Expenditure Weights of the 2019/20-based Consumer Price Indices.". Table 2. https://gia.info.gov.hk/general/202105/28/P2021052800331_368506_1_1622176541579.pdf.

Census and Statistics Department. 2026. "Consumer Price Indices at Commodity/Service Section Level, Table 510-60001A.". Annual 2006 and 2016 observations and October 2017 monthly observations; October 2019–September 2020 base. Source checks dated 17 and 29 September 2026. https://www.censtatd.gov.hk/en/web_table.html?id=510-60001A.

Dustmann, Christian, Bernd Fitzenberger and Markus Zimmermann. 2022. "Housing Expenditure and Income Inequality." *The Economic journal (London)* 132(645):1709–1736.

Epple, Dennis, Luis Quintero and Holger Sieg. 2020. "A new approach to estimating equilibrium models for metropolitan housing markets." *Journal of Political Economy* 128(3):948–983.

Fallis, George and Lawrence B. Smith. 1984. "Uncontrolled Prices in a Controlled Market: The Case of Rent Controls." *The American Economic Review* 74(1):193–200.

Favilukis, Jack, Pierre Mabille and Stijn Van Nieuwerburgh. 2023. "Affordable Housing and City Welfare." *The Review of Economic Studies* 90(1):293–330.

Glaeser, Edward and Joseph Gyourko. 2018. "The economic implications of housing supply." *Journal of economic perspectives* 32(1):3–30.

Glaeser, Edward L. and Erzo F. P. Luttmer. 2003. "The Misallocation of Housing Under Rent Control." *American Economic Review* 93(4):1027–1046.

Gould, J. R. and S. G. B. Henry. 1967. "The Effects of Price Control on a Related Market." *Economica* 34(133):42–49.

Hong Kong Housing Authority. 2016. "Income and Asset Limits for Public Rental Housing 2016/17." Press release, 15 March 2016. https://www.info.gov.hk/gia/general/201603/15/P201603150739.htm.

Hsieh, Chang-Tai and Enrico Moretti. 2019. "Housing constraints and spatial misallocation." *American economic journal: macroeconomics* 11(2):1–39.

Kang, Zi Yang. 2023. The public option and optimal redistribution. Technical report Working paper. January. https://ziyangkang.com/files/jmp.pdf.

Lands Department. 2016. "Land Sale Records, Financial Year 2015/16.". Calendar-2016 awards, cross-checked against government tender-award releases. https://www.landsd.gov.hk/doc/en/landsale/records/2015-2016.pdf.

Lands Department. 2017. "Land Sale Records, Financial Year 2016/17.". Calendar-2016 awards, cross-checked against government tender-award releases. https://www.landsd.gov.hk/doc/en/landsale/records/2016-2017.pdf.

Landvoigt, Tim, Monika Piazzesi and Martin Schneider. 2015. "The Housing Market(s) of San Diego." *American Economic Review* 105(4):1371–1407.

Legislative Council. 2017. "Official Record of Proceedings, 22 February 2017." Hong Kong Legislative Council. Average waiting time for public rental housing, p. 4539. https://www.legco.gov.hk/yr16-17/english/counmtg/hansard/cm20170222-translate-e.pdf.

Leung, Charles Ka Yui, Joe Cho Yiu Ng and Edward Chi Ho Tang. 2020*a*. "What do we know about housing supply? The case of Hong Kong SAR." *Economic and Political Studies* 8(1):6–20.

Leung, Charles Ka Yui, Joe Cho Yiu Ng and Edward Chi Ho Tang. 2020*b*. Why is the Hong Kong housing market unaffordable? Some stylized facts and estimations. ISER Discussion Paper 1081 Institute of Social and Economic Research, Osaka University.

Leung, K. M., L. H. T. Choy and K. W. Chau. 2022. "Examining Informal Housing Supply through Rent Gap Analysis: An Empirical Study of Sub-divided Units in Hong Kong." *Cities* 126:103666. Tables 4–6. https://doi.org/10.1016/j.cities.2022.103666.

Määttänen, Niku and Marko Terviö. 2022. "Welfare Effects of Housing Transaction Taxes: A Quantitative Analysis with an Assignment Model." *The Economic Journal* 132(644):1566–1599.

Mense, Andreas, Claus Michelsen and Konstantin A. Kholodilin. 2023. "Rent Control, Market Segmentation, and Misallocation: Causal Evidence from a Large-Scale Policy Intervention." *Journal of Urban Economics* 134(C).

Monras, Joan and Jose G. Montalvo. 2025. "The Effect of Rent Controls along the "Excess" Price Distribution." Working paper, February. https://jgmontalvo.com/wp/Rent_caps_2025.pdf.

Määttänen, Niku and Marko Terviö. 2014. "Income distribution and housing prices: An assignment model approach." *Journal of Economic Theory* 151:381–410.

Rating and Valuation Department. 2007–2017*a*. "Hong Kong Property Review.". Annual editions. Table 11: new residential units by occupation and area class, preceding calendar year; Table 5 in the 2007 and 2017 editions: private completions. https://www.rvd.gov.hk/en/publications/hkpr.html.

Rating and Valuation Department. 2007, 2017*b*. "Hong Kong Property Review, Table 12: Private Domestic—Average Rents by Class.". January 2006 and June 2016 observations, from the 2007 and 2017 editions; HK dollars per square metre per month, by region and area class. https://www.rvd.gov.hk/doc/en/PR_fullbook/PR2007.pdf, https://www.rvd.gov.hk/doc/en/PR_fullbook/PR2017.pdf.

Taghizadeh-Hesary, Farhad, Naoyuki Yoshino, Aline Mortha, Alvin Chiu and Niki Naderi. 2020. "Internal and External Determinants of Housing Price Boom in Hong Kong." *Buletin Ekonomi Moneter dan Perbankan* 23:597–620.

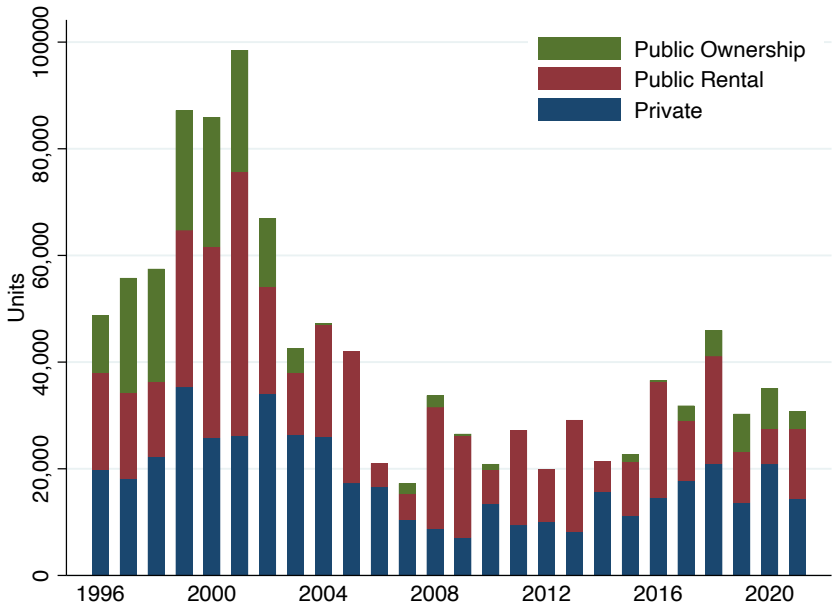
Transport and Housing Bureau. 2018. "LCQ16: Recovery and Allocation of Public Rental Housing Units." Reply to the Legislative Council, 27 June 2018. https://www.info.gov.hk/gia/general/201806/27/P2018062700292.htm.

Wang, Shing-Yi. 2011. "State Misallocation and Housing Prices: Theory and Evidence from China." *American Economic Review* 101(5):2081–2107.

A Supporting Evidence for the Stylized Facts

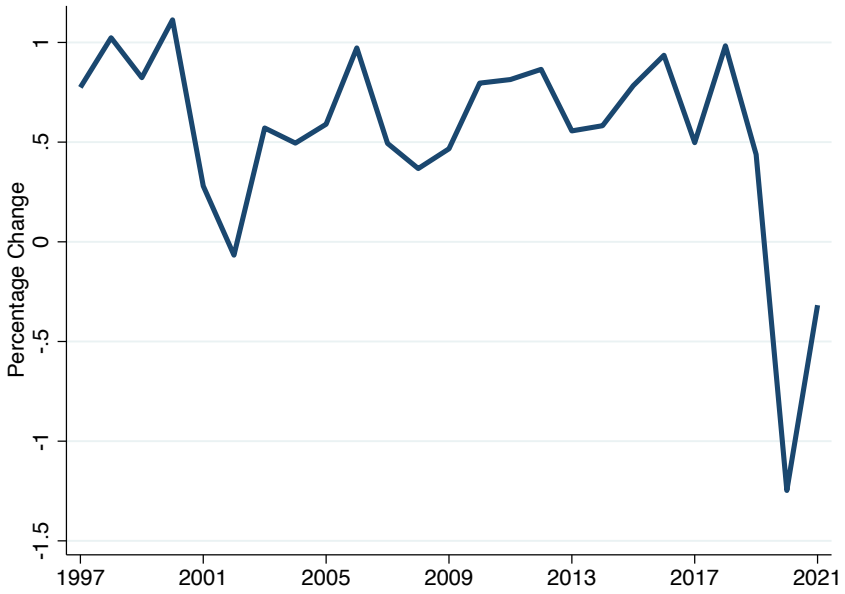
The exhibits below provide background series on construction, population and media coverage; supplementary evidence on price growth by quality; renter and dwelling-size statistics; and illustrations of subdivision. The price figures concern asset prices and price-to-rent ratios, complementing the rental evidence in the main text. The subdivision images are illustrative examples from the dates identified in their source notes, not a representative sample of dwellings in 2006–2016.

Figure A1: Trends in construction



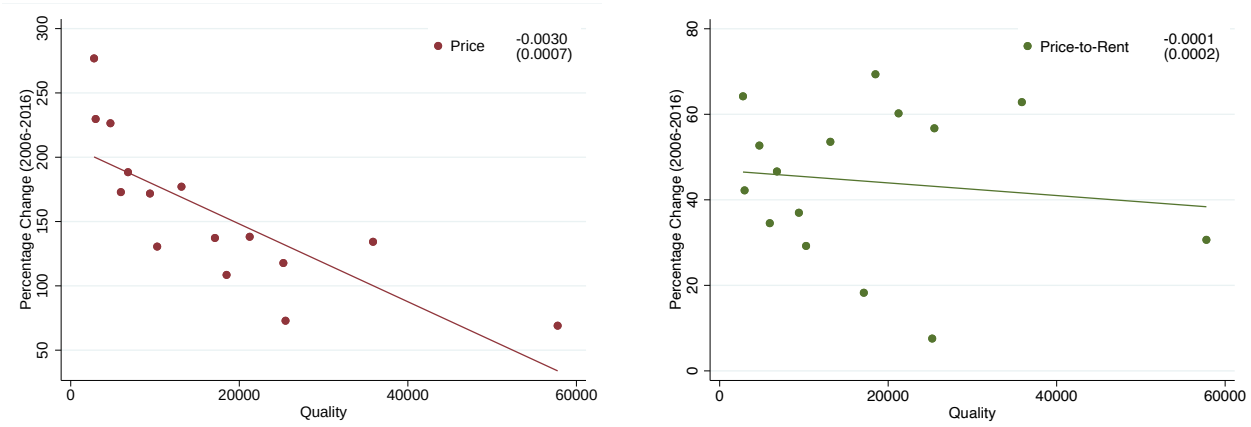
Notes: Figure plots the units of new construction, from 1996 to 2021. Units are divided into three categories: public ownership, public rental, and private.

Figure A2: Population growth, 1997 - 2021



Notes: Figure plots the percentage change in Hong Kong population. Population figures are recorded and reported by the Hong Kong Census and Statistics Department.

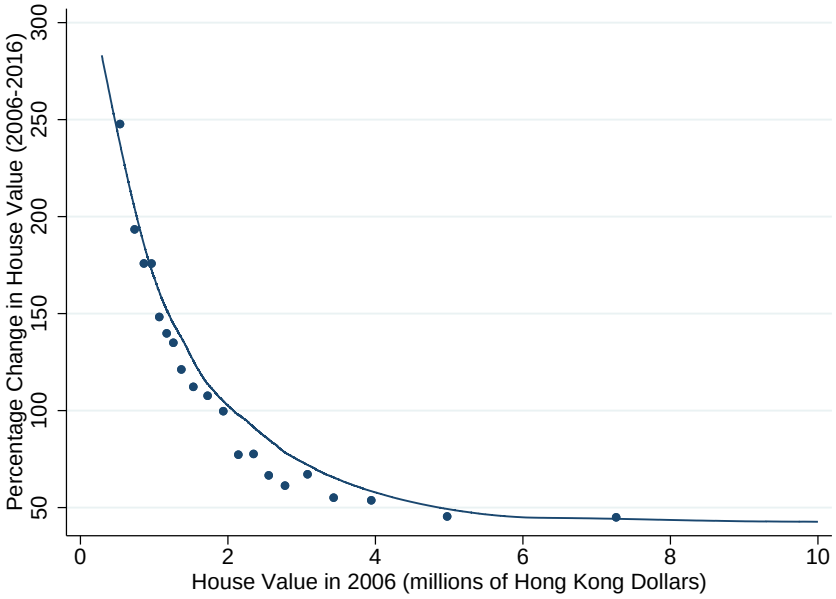
Figure A3: Change in Prices and Price-to-Rent Ratio by Quality Segment



(a) Prices (b) Price-to-rent Ratio

Note: Figure plots percentage change in prices (Panel A) and price-to rent ratio (Panel B) between 2006 and 2016 against unit quality. The price-to-rent ratio is computed as the price index divided by the annualized rent index (i.e., the monthly rent index multiplied by 12). Each dot represents one quality segment, which is comprised of one region and one unit class. Set of regions contains Hong Kong Island, Kowloon and Territories. There are 5 area classes in total, from A to E. The horizontal axis shows the unit quality. Regression coefficients and standard error (in parenthesis) are included in the legend.

Figure A4: Price Growth by Initial Value of Repeat-Sales Units, 2006–2016



Notes: Figure presents a binned scatter plot of the percentage change in transaction prices between 2006 and 2016 against the 2006 transaction price, for units transacted in both years. Data: Midland Realty.

Table A1: Summary Statistics (Renters)

		2001	2006	2011	2016	2021
A.	**Public Renter**					
	Population	2,127,980	2,126,020	2,152,100	2,103,220	2,172,600
	As share of a total population	32.9%	32.0%	30.4%	29.6%	29.9%
	Number of HHs	631,180	696,940	747,940	763,580	768,860
	Share below PRH income limit	56%	56%	62%	62%	63%
	Real HH Income	15,914	14,434	14,093	17,011	17,812
		(11,774)	(11,827)	(10,973)	(13,546)	(13,950)
	Real HH rent	1,425	1,627	1,297	1,401	1,789
		(666)	(752)	(649)	(739)	(830)
	Average HH size	3.4	3.1	2.9	2.8	2.7
	Share with members aged < 15	33.5%	29.6%	23.8%	18.5%	18.4%
	Share with members aged > 60	45.2%	42.1%	46.0%	51.3%	60.6%
	Share moved in last 5 years	29.7%	29.9%	20.2%	15.9%	14.5%
B.	**Private Renter**					
	Population	742,640	702,480	860,860	1,098,700	1,164,520
	As share of a total population	11.5%	10.6%	12.2%	15.5%	16.0%
	Number of HHs	299,600	284,100	332,880	414,900	446,820
	Share below PRH income limit	29%	23%	23%	34%	36%
	Real HH Income	32,491	37,515	45,466	40,437	35,734
		(49,136)	(54,764)	(60,234)	(51,611)	(32,611)
	Real HH rent	7,763	8,080	9,544	10,416	11,086
		(10,706)	(11,103)	(10,896)	(10,040)	(10,539)
	Average HH size	2.5	2.5	2.6	2.6	2.5
	Share with members aged < 15	30.8%	30.3%	32.0%	32.6%	30.1%
	Share with members aged > 60	16.9%	14.6%	15.7%	18.6%	23.7%
	Share moved in last 5 years	75.5%	69.2%	72.5%	64.9%	56.8%

Notes: Table summarizes the summary statistics of private and public renters respectively, using the 5% sample of the Hong Kong Population Census from 2001 to 2021. Standard deviations are reported in parentheses. In both 2006 and 2016, public renters have lower mean household incomes and rents than private renters and are less likely to have moved in the previous five years.

Table A2: Housing Unit Size Distribution: PRH vs. Private Housing

	2005	2008	2011	2014	2017	2020
PRH						
<40 sq m	79.7%	80.3%	81.0%	81.9%	82.6%	83.2%
>40 sq m	20.3%	19.8%	18.9%	18.1%	17.4%	16.7%
Private Housing						
<40 sq m	33.1%	32.4%	31.7%	31.2%	31.2%	31.6%
>40 sq m	66.9%	67.6%	68.3%	68.8%	68.8%	68.4%

Notes: Table summarizes the size distribution of Public Rental Housing (PRH) and private housing units in Hong Kong, categorized by units below and above 40 square meters, for selected years from 2005 to 2020.

Figure A5: Floor plan of subdivided unit



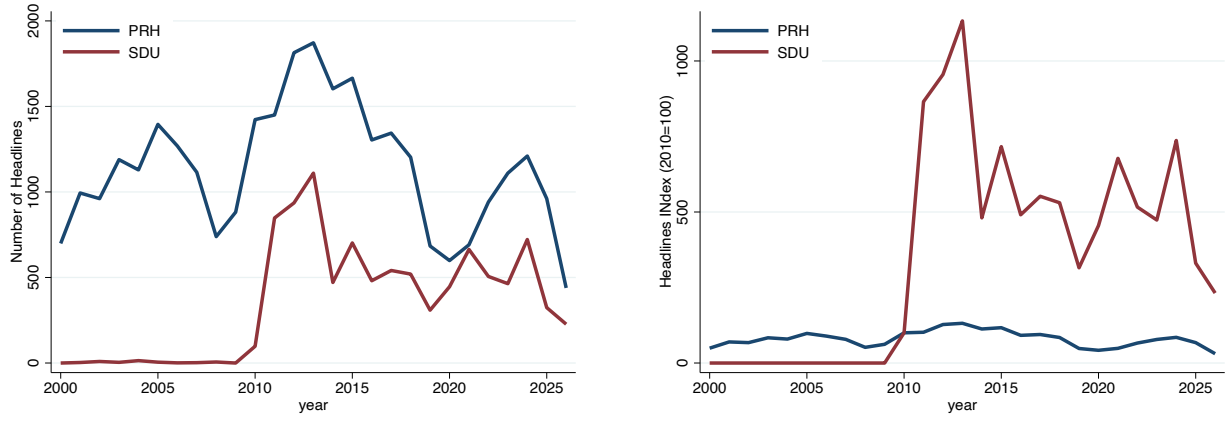
Notes: Figure illustrates the transformation of two 641 sq ft residential units into six subdivided units with floor areas ranging from 126 to 147 sq ft. The "AFTER" layout highlights significant fire and safety hazards. *Source: The South China Morning Post, Naomi Ng, 2017.*

Figure A6: Interior of a subdivided unit



Notes: Huang Chaixiang, 79, poses for a photo at her subdivided flat in Sham Shui Po. This photograph shows the interior of a subdivided unit, revealing the extreme overcrowding and multifunctional use of space. *Source: The South China Morning Post, Eugene Lee, 2024.*

Figure A7: Media Mention of PRH and SDU



(a) Headline Counts (b) Index (2010 = 100)

Notes: Headline mentions of public rental housing (PRH) and subdivided units (SDU) in the Hong Kong Chinese-language newspaper and magazine sample. Panel A reports counts; Panel B indexes each series to 100 in 2010.

B Proofs

B.1 Preliminary

Write $F_C = F(I_{HC})$, $F_L = F(I_{HL})$, $f_C = f(I_{HC})$ and $f_L = f(I_{HL})$. Since $r_C < r_L$, we have $r_H - \delta r_C > r_H - \delta r_L$ and hence $I_{HC} > I_{HL}$ directly from (20) and (15); the maintained condition $nF(I_{HL}) > \mu > 0$ puts I_{HL} strictly above $I_{\min}$, and hence $I_{HC} > I_{HL} > I_{\min}$. Thus $F_C > F_L$ and $f_C, f_L > 0$ by the continuity and positivity of f on its support interior.

It is convenient to summarize the response of rationed L-demand through two reduced-form slopes. Writing D_L as a function of the sorting thresholds via $F_L = F(I_{HL})$ and $F_C = F(I_{HC})$, define

$$\begin{aligned} \omega_L &\equiv f_L \frac{\partial D_L}{\partial F_L} = n f_L \frac{nF_C - \mu}{nF_C - \lambda\mu} > 0, \\ \omega_C &\equiv f_C \frac{\partial D_L}{\partial F_C} = \frac{n(1-\lambda)\mu(nF_L - \lambda\mu)}{(nF_C - \lambda\mu)^2} f_C \geq 0. \end{aligned} \tag{30}$$

Both are nonnegative: $\omega_L > 0$ because $nF_C > \mu$ (the lottery pool is over-subscribed), and $\omega_C \geq 0$ because $nF_L \geq \mu \geq \lambda\mu$, with $\omega_C > 0$ whenever $\lambda < 1$. Through the threshold derivatives (36)–(37), rationed L-demand has partials

$$\frac{\partial D_L}{\partial r_L} = -\frac{\delta}{1-\delta}\,\omega_L, \qquad \frac{\partial D_L}{\partial r_H} = \frac{\omega_L + \omega_C}{1-\delta}, \qquad \frac{\partial D_L}{\partial r_C} = -\frac{\delta}{1-\delta}\,\omega_C. \tag{31}$$

The partial derivatives of the total supply functions with respect to rents, which appear repeatedly in the proofs, follow directly from (9)–(10) and the developer supply in each regime. We adopt the shorthand

$$S_{LL} \equiv \frac{\partial S_L}{\partial r_L}, \quad S_{LH} \equiv \frac{\partial S_L}{\partial r_H}, \quad S_{HL} \equiv \frac{\partial S_H}{\partial r_L}, \quad S_{HH} \equiv \frac{\partial S_H}{\partial r_H},$$

whose explicit values are:

$$S_{LL} = \frac{1}{\gamma} + \mathbf{1}_{\{\rho_H < \rho_L\}} \frac{1}{b\theta_L^2} > 0, \tag{32}$$

$$S_{LH} = -\frac{\theta_L}{\theta_H \gamma} < 0, \tag{33}$$

$$S_{HL} = -\frac{\theta_L}{\theta_H \gamma} < 0, \tag{34}$$

$$S_{HH} = \frac{\theta_L^2}{\theta_H^2 \gamma} + \mathbf{1}_{\{\rho_H > \rho_L\}} \frac{1}{b\theta_H^2} > 0, \tag{35}$$

where $\mathbf{1}_{\{.\}}$ is the indicator for the relevant production regime. These two-variable supply derivatives are used only in the strict active-production regimes. On the co-production locus, supply is a correspondence; we instead use the reduced scalar equation in Lemma 1.

The partial derivatives of I_{HC} and I_{HL} follow from (20) and (15):

$$\frac{\partial I_{HC}}{\partial r_H} = \frac{1}{1-\delta}, \qquad \frac{\partial I_{HC}}{\partial r_C} = -\frac{\delta}{1-\delta}; \tag{36}$$

$$\frac{\partial I_{HL}}{\partial r_H} = \frac{1}{1-\delta}, \qquad \frac{\partial I_{HL}}{\partial r_L} = -\frac{\delta}{1-\delta}. \tag{37}$$

Lemma 1 (Boundary-locus determinacy)**.** *Under the maintained interior-conversion conditions, on the co-production locus $\mathcal{B}$ of (12) the landlord's net conversion is the constant $\Delta^* = \Delta^\dagger$ of (13), so that $h_L^l = h_L^0 + \Delta^\dagger$ and $h_H^l = h_H^0 - \frac{\theta_L}{\theta_H}\Delta^\dagger$ are constant along $\mathcal{B}$ and do not respond to rents there. Treating the production mix $\phi \equiv h_L^n \in [0, \rho^*/(b\theta_L)]$ as an unknown, the market-clearing conditions reduce to a single scalar equation $\Omega(r_L) = 0$ in which Ω is strictly increasing. Consequently the boundary regime is determinate: it contains at most one equilibrium, at which r_L^*, $r_H^* = \Theta r_L^* + \theta_H(\kappa_H - \kappa_L)$, and the mix ϕ are uniquely determined, and this equilibrium is a regular zero of Ω.*

Proof. By (13), $\Delta^* = \Delta^\dagger$ on $\mathcal{B}$, so $h_L^l = h_L^0 + \Delta^\dagger$ and $h_H^l = h_H^0 - \frac{\theta_L}{\theta_H}\Delta^\dagger$ by (9)–(10). Developer supply (11) satisfies $\theta_L\phi + \theta_H h_H^n = \rho^*/b$, $\rho^* = r_L/\theta_L - \kappa_L$. Low-quality clearing $S_L = h_L^l + \phi = D_L$ fixes $\phi = D_L - h_L^l$, where on $\mathcal{B}$ $dI_{HL}/dr_L = (\Theta - \delta)/(1-\delta)$ makes D_L a function of r_L with $dD_L/dr_L \geq 0$. Substituting ϕ and h_H^n into $S_L + S_H = n$, where $\Delta^\dagger$

cancels because $\frac{\theta_L}{\theta_H}h_L^l + h_H^l = \frac{\theta_L}{\theta_H}h_L^0 + h_H^0$, gives

$$\Omega(r_L) \equiv \left(1 - \tfrac{\theta_L}{\theta_H}\right) D_L(r_L) + \frac{r_L/\theta_L - \kappa_L}{b\theta_H} + h_H^0 + \tfrac{\theta_L}{\theta_H}h_L^0 - n = 0, \qquad \Omega'(r_L) \geq \frac{1}{b\theta_H\theta_L} > 0, \tag{38}$$

which has at most one root, a regular zero, with ϕ and r_H^* from (12). Such a root is an equilibrium only if the construction mix also satisfies $0 \leq \phi \leq \rho^*/(b\theta_L)$. □

B.2 Proof of Proposition 1

Sufficient conditions. Write $a \equiv \theta_L/\theta_H \in (0,1)$ and $\Delta^\dagger = \theta_L(\kappa_L - \kappa_H)/\gamma$. We take $h_L^0, h_H^0 > 0$ and require

$$n \geq h_L^0 + h_H^0, \qquad -h_L^0 < \Delta^\dagger < a^{-1}h_H^0. \tag{Stock$'$}$$

Define the supply objects Q, R_L, R_H in Step 1 and let

$$X \equiv \min\left\{n, \frac{\theta_H n - \theta_L h_L^0 - \theta_H h_H^0}{\theta_H - \theta_L}\right\}, \qquad x_a \equiv \min\{x \in [0, X] : R_L(x) \geq 0\}. \tag{39}$$

Our remaining sufficient conditions are

$$R_L(X) > 0, \qquad R_L(x) < I_{\min} \quad \text{for all } x \in [x_a, X], \tag{Afford$'$}$$

$$F\left(\frac{R_H(x_a) - \delta R_L(x_a)}{1-\delta}\right) > \frac{x_a}{n}. \tag{Sort$'$}$$

If $R_L(0) \geq 0$, then $x_a = 0$ and (Sort$'$) reduces to $I_{HL}(0) > I_{\min}$. If $R_L(0) < 0$, sorting is instead checked at the first nonnegative L rent, where $R_L(x_a) = 0$.

Step 1: Supply and supporting rents. Let x denote total L housing. Land conservation gives

$$Q(x) = \theta_H n - (\theta_H - \theta_L)x - \theta_L h_L^0 - \theta_H h_H^0. \tag{40}$$

Thus feasible construction requires $x \in [0, X]$, with $Q(x) > 0$ for $x < X$. If $X < n$, then $Q(X) = 0$; $Q(X) = 0$ is also possible when $X = n$. Set $p(x) = bQ(x)$. For $0 < x < X$, the production regimes are

$$\begin{aligned} H\text{-only}: &\quad x < h_L^0 + \Delta^\dagger, \\ \text{co-production}: &\quad h_L^0 + \Delta^\dagger \le x \le n - h_H^0 + a\Delta^\dagger, \\ L\text{-only}: &\quad x > n - h_H^0 + a\Delta^\dagger. \end{aligned}$$

These intervals are intersected with $(0, X)$; an interval can be empty. Feasible co-production requires $n - h_L^0 - h_H^0 - (1-a)\Delta^\dagger \ge 0$; if this fails, X lies in the H-only range and the other two ranges are empty.

At any equilibrium with $0 < x < n$, conversion is interior, including when construction is zero. At its lower physical bound, either no L is built, so $x = 0$, or free entry gives $r_L = k_L + p\theta_L$ and $r_H \le k_H + p\theta_H$, implying $r_L - ar_H \ge k_L - ak_H = \gamma\Delta^\dagger > -\gamma h_L^0$, contrary to landlord optimality. The upper-bound argument is symmetric: either $x = n$, or H is built and $r_L - ar_H \le \gamma\Delta^\dagger < \gamma a^{-1} h_H^0$. For positive construction, the landlord condition and free entry imply

$$\Delta(x) = \begin{cases} x - h_L^0, & H\text{-only}, \\ \Delta^\dagger, & \text{co-production}, \\ a^{-1}(x - n + h_H^0), & L\text{-only}, \end{cases} \tag{41}$$

$$\begin{aligned} R_L(x) &= \begin{cases} ak_H + \theta_L p(x) + \gamma(x - h_L^0), & H\text{-only}, \\ k_L + \theta_L p(x), & \text{otherwise}, \end{cases} \\ R_H(x) &= \begin{cases} k_H + \theta_H p(x), & H\text{-only or co-production}, \\ a^{-1}k_L + \theta_H p(x) - \gamma a^{-2}(x - n + h_H^0), & L\text{-only}. \end{cases} \end{aligned} \tag{42}$$

Use continuous limits at $x = 0, X$. The quantities $h_L^n = x - h_L^0 - \Delta(x)$ and $h_H^n = n - x - h_H^0 + a\Delta(x)$ are nonnegative and use $Q(x)$ land. The schedules are continuous, satisfy $R_L - aR_H = \gamma\Delta(x)$, and support landlord optimality and free entry. R_L is concave: its slope is $\gamma - b\theta_L(\theta_H - \theta_L)$ in the H-only range and $-b\theta_L(\theta_H - \theta_L)$ thereafter. R_H strictly decreases

in all positive-construction ranges.

If $X = n$, then $R_L(X) = k_L + \theta_L bQ(n)$. If $X < n$, define

$$\Delta_X \equiv \frac{n - h_L^0 - h_H^0}{1 - a} \geq 0. \tag{43}$$

Here $0 < X < n$, $\Delta(X) = \Delta_X$, and

$$R_L(X) = \min\{k_L, ak_H + \gamma\Delta_X\}, \qquad R_H(X) = \frac{R_L(X) - \gamma\Delta_X}{a}. \tag{44}$$

The limiting rents satisfy $R_L(X) \leq k_L$, $R_H(X) \leq k_H$. A no-construction equilibrium must lie at $x = X$, with $p = 0$, $r_L - ar_H = \gamma\Delta_X$ and $r_j \leq k_j$.

Step 2: A strictly monotone sorting equation. Let $V(x) = R_H(x) - \delta R_L(x)$ and $I_{HL}(x) = V(x)/(1 - \delta)$. The schedules are continuous, with slopes

$$V'(x) = \begin{cases} -b(\theta_H - \delta\theta_L)(\theta_H - \theta_L) - \delta\gamma, & H\text{-only}, \\ -b(\theta_H - \delta\theta_L)(\theta_H - \theta_L), & \text{co-production}, \\ -b(\theta_H - \delta\theta_L)(\theta_H - \theta_L) - \gamma a^{-2}, & L\text{-only}. \end{cases} \tag{45}$$

All slopes are strictly negative. Sorting with positive construction requires

$$V(x) = (1 - \delta)F^{-1}(x/n), \qquad 0 < x < X. \tag{46}$$

The left side strictly decreases and the right side strictly increases. Condition (Sort′) places the cutoff strictly above the income quantile at x_a (using $F^{-1}(0) = I_{\min}$). Since $X > 0$ and $R_L(X) > 0$, continuity gives $x_a < X$. Concavity and $R_L(X) > 0$ give $R_L(x) > 0$ for $x_a < x \leq X$. A feasible equilibrium cannot lie to the left of x_a because rents there are negative. If $X = n$, the income quantile diverges as $x \uparrow n$, so there is exactly one crossing before X.

Step 3: Construction or no construction. If the schedules cross at $x^* \in (x_a, X)$, Steps 1–2 give the unique positive-construction equilibrium. At this crossing, $0 < r_L < I_{\min}$

and

$$r_H = \delta r_L + (1-\delta)F^{-1}(x^*/n).$$

Since $F^{-1}(x^*/n) > I_{\min} > r_L$, we obtain $r_L < r_H < F^{-1}(x^*/n)$. Every household choosing H can therefore afford it.

Otherwise $X < n$, $Q(X) = 0$, and $I_{HL}(X) \geq T_X \equiv F^{-1}(X/n) > I_{\min}$. The no-construction equilibrium has $x = X$, $\Delta = \Delta_X$ and

$$r_L = \frac{\gamma\Delta_X + a(1-\delta)T_X}{1-a\delta}, \qquad r_H = \frac{\delta\gamma\Delta_X + (1-\delta)T_X}{1-a\delta}. \tag{47}$$

Both rents are strictly positive. Along the line $r_L - ar_H = \gamma\Delta_X$, the cutoff strictly increases with either rent. Consequently $I_{HL}(X) \geq T_X$ implies $r_j \leq R_j(X) \leq k_j$, so no developer enters, and $r_L \leq R_L(X) < I_{\min}$. Sorting again gives $r_L < r_H < T_X$.

There is no other no-construction equilibrium: land conservation fixes $x = X$, and the landlord and sorting equations uniquely determine (47). If a crossing occurs before X, strict monotonicity gives $I_{HL}(X) < T_X$, so the required no-construction rents exceed the limiting rents and violate at least one free-entry inequality. An all-L equilibrium is impossible because the income support is unbounded and sufficiently rich households prefer H at finite rents. An all-H equilibrium has positive construction because $Q(0) > 0$. It requires $r_H = R_H(0)$ and $r_L \leq R_L(0)$. If $R_L(0) < 0$, this contradicts nonnegative rents. Otherwise $x_a = 0$ and (Sort$'$) gives $I_{HL}(0) > I_{\min}$, implying positive L demand (or infeasibility of housing all households in H). Thus both qualities are occupied, and rents, conversion and aggregate construction are unique. □

B.3 Non-market existence for every targeting share

Put $M = n - \mu$ and construct Q, R_L, R_H, X from Step 1 using M in place of n, while retaining F as the income distribution of all households. Let

$$A = \{x \in [0, X] : R_L(x) > r_C\}, \qquad x_\ell = \inf A, \qquad I_{HC}(x) = \frac{R_H(x) - \delta r_C}{1-\delta}.$$

Assume A is nonempty. It is an interval because R_L is concave; cutoffs at x_ℓ are evaluated by continuous limits.

Corollary 1. *Suppose* $0 < \mu < n$, $r_C \geq 0$, $M \geq h_L^0 + h_H^0$, $-h_L^0 < \Delta^\dagger < a^{-1}h_H^0$, *and* $R_L(x) < I_{\min}$ *on* A. *If*

$$nF(I_{HL}(x_\ell)) > \mu + x_\ell, \tag{NM1}$$

and, for some $\bar{x} \in A$ *with* $\bar{x} < X$,

$$0 < nF(I_{HC}(\bar{x})) - \mu < \bar{x}, \tag{NM2}$$

then, for every $\lambda \in [0,1]$, *there is exactly one equilibrium with all* μ *non-market places occupied and* $r_C < r_L < I_{\min}$, $r_L < r_H$. *It has positive construction and both private qualities are occupied.*

Proof. Write $F_L = F(I_{HL})$ and $F_C = F(I_{HC})$. When $nF_C > \mu$, the paper's uniform targeting and residual lottery rule gives the general demand

$$D_L = \frac{nF_C - \mu}{nF_C - \lambda\mu}\left[nF_L - \lambda \min\{nF_L, \mu\}\right]. \tag{48}$$

Set $D_L = 0$ when $nF_C = \mu$. This expression is continuous on the filled-capacity domain $nF_C \geq \mu$, weakly increasing in both cutoffs, and satisfies $(nF_L - \mu)_+ \leq D_L \leq nF_C - \mu$. When $nF_L \geq \mu$, it reduces to (24). Both cutoffs strictly decrease with x along the supply schedules, so $D_L(x) - x$ strictly decreases on the part of A with $nF_C \geq \mu$. By (NM1), it is positive just to the right of x_ℓ; by (NM2), it is negative at $\bar{x}$. There is therefore a unique root $x^* \in (x_\ell, \bar{x})$, with $nF_C > \mu$, $r_C < R_L(x^*) < I_{\min}$ and $Q(x^*) > 0$. Since $D_L < nF_L$ and $r_L < I_{\min}$, the root has $I_{HL} > I_{\min}$; the cutoff equation then gives $r_H > r_L$ and affordability for all private H renters. Also $D_L \leq nF_C - \mu < M$, so both qualities are occupied.

A no-construction equilibrium would have $x = X < M$. Its landlord condition and free-entry inequalities imply $r_j \leq R_j(X)$, hence both cutoffs would be strictly below those at $x^* < X$. If all places were filled, demand monotonicity would give $D_L \leq D_L(x^*) = x^* < X$, a contradiction. An all-H equilibrium requires $r_H = R_H(0)$ and $r_L \leq R_L(0)$. If $R_L(0) \leq r_C$, it violates $r_L > r_C$; otherwise $x_\ell = 0$ and (NM1) implies positive private L demand. An all-L

equilibrium is impossible since $D_L \leq nF_C - \mu < M$ at finite rents. □

Propositions 2–3 apply when the equilibrium satisfies (21), with (22) for case (iii).

B.4 Proof of Proposition 2

Holding G fixed, we differentiate (26) and (27) with respect to r_C, then apply Cramer's rule. We first derive the demand derivative common to both regimes, then treat each regime in turn.

Preliminary: derivative of market L-demand. By the reduced-form partials (31), the total derivative of market L-demand along a rent path indexed by r_C is

$$\frac{dD_L}{dr_C} = -\frac{\delta}{1-\delta}\omega_L \frac{dr_L}{dr_C} + \frac{\omega_L + \omega_C}{1-\delta}\frac{dr_H}{dr_C} - \frac{\delta}{1-\delta}\omega_C, \tag{49}$$

where the last term is the direct effect of r_C through I_{HC}.

Case (i): High-quality production regime. *Aggregate clearing* (27). Setting $d(S_L + S_H)/dr_C = 0$:

$$(S_{LL} + S_{HL})\frac{dr_L}{dr_C} + (S_{LH} + S_{HH})\frac{dr_H}{dr_C} = 0. \tag{50}$$

Low-quality clearing (26). Setting $dS_L/dr_C = dD_L/dr_C$ and using (49):

$$\left(S_{LL} + \frac{\delta}{1-\delta}\omega_L\right)\frac{dr_L}{dr_C} - \left(|S_{LH}| + \frac{\omega_L + \omega_C}{1-\delta}\right)\frac{dr_H}{dr_C} = -\frac{\delta}{1-\delta}\omega_C. \tag{51}$$

Linear system. Let $b_1 \equiv -\dfrac{\delta}{1-\delta}\omega_C \leq 0$ (strictly negative for $\lambda < 1$). Equations (51) and

(50) form the system $A_H(dr_L/dr_C,\, dr_H/dr_C)^\top = (b_1, 0)^\top$, with entries

$$\begin{aligned}
a_{11} &= S_{LL} + \frac{\delta}{1-\delta}\,\omega_L, \\
a_{12} &= -|S_{LH}| - \frac{\omega_L + \omega_C}{1-\delta}, \\
a_{21} &= S_{LL} + S_{HL} = \left(1 - \frac{\theta_L}{\theta_H}\right)\frac{1}{\gamma}, \\
a_{22} &= S_{LH} + S_{HH} = \frac{\gamma - b\theta_L(\theta_H - \theta_L)}{b\gamma\theta_H^2}.
\end{aligned}$$

Determinant. Expanding $\det(A_H) = a_{11}a_{22} - a_{12}a_{21}$:

$$\det(A_H) = \frac{1}{b\gamma\theta_H^2} + \frac{\omega_L}{1-\delta}\left[\frac{\delta}{b\theta_H^2} + \frac{1}{\gamma}\left(1 - \frac{\theta_L}{\theta_H}\right)\left(1 - \frac{\delta\theta_L}{\theta_H}\right)\right] + \frac{\omega_C}{\gamma(1-\delta)}\left(1 - \frac{\theta_L}{\theta_H}\right) > 0. \quad (52)$$

The constant term is strictly positive and the other terms are nonnegative since $\delta \in (0,1)$, $\theta_H > \theta_L$, $\omega_L > 0$, and $\omega_C \geq 0$.

Cramer's rule. By Cramer's rule, $dr_j/dr_C = \det(A_{H,j})/\det(A_H)$, where $A_{H,j}$ replaces column j by $(b_1, 0)^\top$:

$$\det(A_{H,1}) = b_1 \cdot a_{22} = -\frac{\delta\,\omega_C\big[\gamma - b\theta_L(\theta_H - \theta_L)\big]}{b\gamma\theta_H^2(1-\delta)}, \quad (53)$$

$$\det(A_{H,2}) = -b_1 \cdot a_{21} = \frac{\delta\,\omega_C}{(1-\delta)\gamma}\left(1 - \frac{\theta_L}{\theta_H}\right) \geq 0. \quad (54)$$

Signs. Take $\lambda < 1$, so $\omega_C > 0$. Since $\det(A_H) > 0$ and $\det(A_{H,2}) > 0$, we have $dr_H/dr_C > 0$. In (53), the factor $-\delta\omega_C/(b\gamma\theta_H^2(1-\delta)) < 0$, so $\det(A_{H,1})$ has the opposite sign to $[\gamma - b\theta_L(\theta_H - \theta_L)]$. Hence $\det(A_{H,1}) < 0$, implying $dr_L/dr_C < 0$, if and only if $\gamma > b\theta_L(\theta_H - \theta_L)$, i.e., (28). This establishes part (i).

Case (ii): Low-quality production regime. *Supply derivatives.* In this regime $S_{LL} = 1/\gamma + 1/(b\theta_L^2)$ (entry adds to L-supply) and $S_{HH} = \theta_L^2/(\theta_H^2\gamma)$; S_{LH} and S_{HL} are unchanged.

Aggregate clearing (27):

$$(S_{LL} + S_{HL})\frac{dr_L}{dr_C} + (S_{LH} + S_{HH})\frac{dr_H}{dr_C} = 0. \quad (55)$$

Low-quality clearing (26): identical in form to (51), with S_{LL} now taking its low-quality regime value.

Linear system. Equations give the system $A_L(dr_L/dr_C,\, dr_H/dr_C)^\top = (b_1, 0)^\top$ with $b_1 < 0$ as before and entries

$$\begin{aligned}
a_{11} &= S_{LL} + \frac{\delta}{1-\delta}\,\omega_L, \quad \text{with } S_{LL} = \frac{1}{\gamma} + \frac{1}{b\theta_L^2},\\
a_{12} &= -|S_{LH}| - \frac{\omega_L + \omega_C}{1-\delta},\\
a_{21} &= S_{LL} + S_{HL} = \left(1 - \frac{\theta_L}{\theta_H}\right)\frac{1}{\gamma} + \frac{1}{b\theta_L^2},\\
a_{22} &= S_{LH} + S_{HH} = -\frac{\theta_L}{\theta_H \gamma}\left(1 - \frac{\theta_L}{\theta_H}\right).
\end{aligned}$$

Determinant. Expanding and simplifying:

$$\begin{aligned}
\det(A_L) = \frac{1}{b\gamma\theta_H^2}
&+ \frac{\omega_L}{1-\delta}\left[\frac{(1-\frac{\theta_L}{\theta_H})(1-\delta\frac{\theta_L}{\theta_H})}{\gamma} + \frac{1}{b\theta_L^2}\right]\\
&+ \frac{\omega_C}{1-\delta}\left[\frac{1-\frac{\theta_L}{\theta_H}}{\gamma} + \frac{1}{b\theta_L^2}\right] > 0.
\end{aligned} \tag{56}$$

Cramer's rule.

$$\det(A_{L,1}) = b_1 \cdot a_{22} = \frac{\delta\,\omega_C\,\theta_L(1-\theta_L/\theta_H)}{\gamma\theta_H(1-\delta)} \geq 0, \tag{57}$$

$$\det(A_{L,2}) = -b_1 \cdot a_{21} = \frac{\delta\,\omega_C}{1-\delta}\left[\frac{1}{\gamma}\left(1 - \frac{\theta_L}{\theta_H}\right) + \frac{1}{b\theta_L^2}\right] \geq 0. \tag{58}$$

Signs. For $\lambda < 1$, $\det(A_L)$, $\det(A_{L,1})$, and $\det(A_{L,2})$ are all strictly positive, so $dr_L/dr_C > 0$ and $dr_H/dr_C > 0$: a decrease in r_C lowers both market rents. This establishes part (ii).

Case (iii). Assume the strict construction-mix interior (22). By Lemma 1, on the co-production locus $\mathcal{B}$ the landlord's net conversion is the constant $\Delta^\dagger$ of (13), so its supply

does not respond to rents there, and (26)–(27) reduce to the scalar equation

$$\Omega(r_L; r_C) \equiv \left(1 - \tfrac{\theta_L}{\theta_H}\right) D_L + \frac{r_L/\theta_L - \kappa_L}{b\theta_H} + h_H^0 + \tfrac{\theta_L}{\theta_H} h_L^0 - (n - \mu) = 0, \tag{59}$$

Here D_L is evaluated after substituting $r_H = \Theta r_L + \theta_H(\kappa_H - \kappa_L)$; derivatives with respect to r_L in Ω are derivatives of this reduced function. The constant $\Delta^\dagger$ cancels from the land-weighted aggregate as in Lemma 1, with construction mix $\phi \equiv h_L^n = D_L - h_L^0 - \Delta^\dagger$ and

$$\frac{\partial \Omega}{\partial r_L} = \left(1 - \tfrac{\theta_L}{\theta_H}\right) \frac{(\Theta - \delta)\omega_L + \Theta\omega_C}{1 - \delta} + \frac{1}{b\theta_H\theta_L} > 0.$$

Since r_C enters only through I_{HC}, (31) gives

$$\frac{\partial \Omega}{\partial r_C} = \left(1 - \tfrac{\theta_L}{\theta_H}\right) \frac{\partial D_L}{\partial r_C} = -\left(1 - \tfrac{\theta_L}{\theta_H}\right) \frac{\delta}{1 - \delta}\, \omega_C < 0$$

(for $\lambda < 1$). By the implicit function theorem, and using $dr_H/dr_L = \Theta$ along $\mathcal{B}$ by (12),

$$\frac{dr_L}{dr_C} = -\frac{\partial \Omega/\partial r_C}{\partial \Omega/\partial r_L} > 0, \qquad \frac{dr_H}{dr_C} = \Theta\, \frac{dr_L}{dr_C} > 0,$$

so a decrease in r_C lowers both rents, moving the equilibrium along $\mathcal{B}$ with $dr_H = \Theta\, dr_L$. Differentiating (59), the mix satisfies

$$\frac{d\phi}{dr_C} = \frac{dD_L}{dr_C} = -\frac{1}{\left(1 - \theta_L/\theta_H\right) b\theta_H\theta_L}\, \frac{dr_L}{dr_C} < 0, \tag{60}$$

so low-quality construction rises as r_C falls. Net conversion remains at the constant $\Delta^\dagger = \theta_L(\kappa_L - \kappa_H)/\gamma$, which is non-zero whenever $\kappa_L \neq \kappa_H$.

Perfect targeting. At $\lambda = 1$, $\omega_C = 0$ and $D_L = nF_L - \mu$ is independent of r_C. The right-hand side of either strict-regime system is zero; its positive determinant implies both rent derivatives are zero. In the co-production interior, $\partial\Omega/\partial r_C = 0$ and the positive reduced derivative gives the same result.

B.5 Proof of Proposition 3

We differentiate (26)–(27) with respect to μ, holding G, r_C, h_H^0 fixed with $\partial h_L^0/\partial \mu = -1$.

Direct policy effect. Holding rents fixed, the net change in low-quality excess demand combines $\partial D_L/\partial \mu$ with the stock withdrawal $\partial S_L/\partial \mu = -1$:

$$c_1 \equiv 1 + \left.\frac{dD_L}{d\mu}\right|_{r_L, r_H \text{ fixed}} = \frac{n^2(1-\lambda)F_C(F_C - F_L)}{(nF_C - \lambda\mu)^2} > 0, \tag{61}$$

which is strictly positive since $F_C > F_L$ (as $I_{HC} > I_{HL}$ and F is strictly increasing) and $\lambda < 1$.

In aggregate clearing, the market-stock reduction and the decline in $n - \mu$ cancel. Differentiating low-quality clearing therefore gives the same coefficient matrices as in Appendix B.4, with right-hand side $(c_1, 0)^\top$. Cramer's rule gives, in either strict regime,

$$\frac{dr_L}{d\mu} = \frac{c_1 a_{22}}{\det A}, \qquad \frac{dr_H}{d\mu} = -\frac{c_1 a_{21}}{\det A},$$

where A is the corresponding matrix A_H or A_L.

Case (i): High-quality production regime. Here $\det A_H > 0$ and $a_{21} > 0$, so $dr_H/d\mu < 0$. The sign of a_{22} is the sign of $\gamma - b\theta_L(\theta_H - \theta_L)$; hence $dr_L/d\mu > 0$ if and only if (28) holds.

Case (ii): Low-quality production regime. Here $\det A_L > 0$, $a_{21} > 0$ and $a_{22} < 0$, so both rent derivatives are negative.

Case (iii). Assume the strict construction-mix interior (22). By Lemma 1, on the co-production locus $\mathcal{B}$ clearing reduces to (59) with the μ-dependence made explicit,

$$\Omega(r_L; \mu) \equiv \left(1 - \tfrac{\theta_L}{\theta_H}\right) D_L(r_L; \mu) + \frac{r_L/\theta_L - \kappa_L}{b\theta_H} + h_H^0 + \tfrac{\theta_L}{\theta_H} h_L^0(\mu) - (n - \mu) = 0,$$

where $\partial h_L^0/\partial\mu = -1$ (Section 5.4). Then $\partial\Omega/\partial r_L > 0$ as in Case (iii) of Appendix B.4, and by (61),

$$\frac{\partial\Omega}{\partial\mu} = \left(1 - \tfrac{\theta_L}{\theta_H}\right)\left(1 + \tfrac{dD_L}{d\mu}\Big|_{\text{fixed}}\right) = \left(1 - \tfrac{\theta_L}{\theta_H}\right) c_1 > 0,$$

with $c_1 > 0$ from (61). Hence

$$\frac{dr_L}{d\mu} = -\frac{\partial\Omega/\partial\mu}{\partial\Omega/\partial r_L} < 0, \qquad \frac{dr_H}{d\mu} = \Theta\,\frac{dr_L}{d\mu} < 0,$$

so an increase in μ lowers both rents, moving the equilibrium along $\mathcal{B}$ with $dr_H = \Theta\, dr_L$.

Perfect targeting. At $\lambda = 1$, (24) reduces to $D_L = nF_L - \mu$. The direct demand effect is -1, exactly matching $\partial S_L/\partial\mu = -1$, while aggregate demand and supply also fall by the same amount. Thus $c_1 = 0$. Since the strict-regime coefficient matrices are nonsingular, both rent derivatives are zero. In the co-production interior the reduced equation has $\partial\Omega/\partial\mu = 0$ and $\partial\Omega/\partial r_L > 0$, giving the same zero derivatives.

C Quantitative Appendix

C.1 Sample and Measurement

The sample restrictions and quality categories are described in Section 6. This appendix records the measurement and implementation details. We retain records with positive income and rent; the 2016 income cutoff carries the 2006 cutoff forward along the rent–quality schedule. Household income is the census variable DJHHINC, not corrected for top-coding. Live-in domestic helpers and other non-family members are not counted in the equivalized size s, but their earnings stay in household income. We deflate monetary values with a fixed-weight nonhousing price index built from eight sections of the Composite CPI (Census and Statistics Department, 2021, 2026). The 2016 deflator is 0.795188. The sample represents $N = 741{,}178.14$ tenant households: the official 2016 count of renting households, 1,178,999, times the sample's share of the 58,282 census tenant records.

A private dwelling's quality is inferred as its rent at 2006 prices and market conditions. For 2016 dwellings, we remove rent growth using region and floor-area-class indices, with a fitted rent–quality schedule where a usable cell is unavailable. The quality cutoff is HK$4,800, the median rent among all 2006 private tenants. Dwellings above the cutoff are classified as H, and the rest as L; public flats are treated as L. Representative qualities are the medians on either side of the cutoff, $L = 3{,}000$ and $H = 8{,}000$. Mean floor areas in the 2016 sample are $\theta_L = 29.175$ and $\theta_H = 64.513$ square metres. Comparing stocks across years uses this fixed quality definition and assumes that the inferred qualities and their representative floor areas are comparable.

Household housing choices use total income I. For targeting and distributional comparisons we use equivalized income $y = I/s$, which adjusts for household size and composition.[16] Public housing accommodates 53.07% of sample households and its median rent is HK$1,298.54. Among private tenants, 47.97% occupy H dwellings. Public occupancy by income group and private-quality choices provide the moments used to fit demand and allocation. Appendix Table A3 reports public occupancy under alternative income rankings and household or person

[16] We use $s = 1 + 0.5(\text{adults} - 1) + 0.3\,\text{children}$, counting members aged 14 or over as adults (Dustmann, Fitzenberger and Zimmermann, 2022).

weights. For example, Q1 occupancy is 65.55% when households are ranked by total income and 72.38% when ranked by equivalized income; with person quintiles and person weights it is 73.63%. These distinctions matter because the calibration uses household moments, while the main welfare results give each person equal weight.

Reported incomes bunch at round numbers: 35% of sample households report a multiple of HK$1,000, and 0.67% report exactly HK$50,000, close to I_{HC}. A model cutoff then sticks at a bunch, and rents jump when it crosses one. We smooth the income distribution by treating each household's income as uniform on a band of 5% on either side of the reported value, bounded below by the lowest sample income. Ranks and data moments do not change. Using bands of 2.5% or 10% changes the reported counterfactual rents by less than HK$5 per month. With reported incomes the direction is the same, and L rent falls by 2.2% for $r_C + 50\%$ and by 1.7% for $\mu - 10\%$.

To implement the fixed quality definition, we divide 2016 private tenants' nominal rents by January 2006–June 2016 rent growth in the tenant's region and floor-area class (Rating and Valuation Department, 2007, 2017*b*, Table 12), with the fitted rent–quality schedule for tenants without a usable cell. The reference sample for the quality cutoff and representative qualities comprises 14,240 private tenant households in 2006, including all ages. Floor areas θ_j are group means of floor-area class midpoints in the 2016 sample.

C.2 Construction, Land Costs and the Construction Proxy

The construction charge uses the midpoint of the handbook's range for average-standard high-rise apartments, HK$26,100 per square metre of construction floor area (Arcadis, 2018), with a 2% real discount rate and a 50-year life. We move the quote from late 2017 to mid-2016 with the handbook's tender price index (Arcadis, 2018, p. 49) and use a saleable-to-construction ratio of 0.75.

The baseline land charge p_0 uses the 18 pure-residential government tender awards of 2016, HK$68,024.35 per square metre of permitted gross floor area (Lands Department, 2016, 2017). We convert it to saleable area with a ratio of 0.8 and to a monthly flow at the 2% real rate, treating land as keeping its real value.

The 2% rate is the round number at which zero profit comes closest to the H rent in the

Table A3: Public occupancy by income quintile, 2016

	Total income	Equivalized income		Mean
Group	households	households	persons	size
Q1	65.55	72.38	73.63	1.98
Q2	60.28	65.56	68.23	1.84
Q3	64.20	66.21	71.66	1.85
Q4	56.47	52.51	62.33	1.82
Q5	18.87	8.72	13.32	1.62
Poorest μ	62.97	68.24	71.07	—
All	53.07	53.07	57.83	1.82

Notes: Public occupancy is the percentage of the group living in public rental housing. Quintiles split the 36,639 sample households, or their 103,085 family members, into equal fifths by the ranking variable; ties at a boundary are split fractionally. Mean size is the mean of s across households in the equivalized-income household quintile. In the row for the poorest μ, the group is the poorest μ of households or persons.

data, and Table A11 shows 2.5% and 3%, where the fit is much worse. At 1.5% the fitted L rent reaches the cost of building L, so the market leaves the regime with construction of H only.

The citywide construction proxy multiplies 2006–2016 private completions by the rental share among newly assessed units (Rating and Valuation Department, 2007–2017*a*), which sums to 44,506.25 dwellings. We assume all of it is high quality.[17] We assign to the age 20–60 market its share of all private tenants, 17,193 of 19,519, which gives $T = 39{,}202.62$, or $\tau = T/N = 0.052892$. We treat that market as separate once T is assigned: the model has no competition across age groups for these dwellings. The costs k_j have no separate location premium, since p_0 is an average land charge.

C.3 Historical Mapping for the Benchmark Input-Supply Slope

Rents. The 2006 rents of the two representative dwellings are L and H by construction. Their 2016 rents are the targets of Table A7. We hold the representative floor areas θ_L and θ_H

[17] As descriptive support, 98 of 99 agency-reported private leases on Property.hk in June 2016, in buildings no more than ten years old, have quality above HK$4,800. These geographically concentrated platform records cover leasing rather than first lettings, so all-H construction remains an approximation. The match between the growth of the H stock and T in Table A6 points the same way.

Table A4: Calibrated inputs

Quantity	Value	Source or role
L, H	3,000; 8,000	2006 private tenants, all ages: median rent on each side of the median
N	741,178.14	Tenant households represented by the sample
T	39,202.62	Rental-construction proxy assigned to the sample
k_L, k_H	2,189.38; 4,841.27	Monthly construction capital charges
p_0	112.692	Monthly land charge per saleable m^2, 2016 land sales
$\bar{p}$	81.27	Land rent at no new construction, $p_0 - b\theta_H\tau$
h_L^0, h_H^0	0.2457; 0.1725	Inherited stocks, shares of sample households

Notes: Population is normalized to one in the model. Monetary values are monthly 2006 HK dollars, except quality units, which are defined by 2006 rents. Costs k_j are per dwelling; p_0 and $\bar{p}$ are per square metre.

fixed across dates. This is a maintained comparability assumption, rather than a separate observation of 2006 floor areas.

Land rent. Zero profit on H gives $p_t = r_{H,t}/\theta_H - \kappa_t$. We set $\kappa_{2016} = 75.04$ using Appendix C.2. To get κ_{2006} we move the handbook's building and services costs back to mid-2006 with the tender price indices for public works (Architectural Services Department, 2024), which gives a real ratio $\kappa_{2006}/\kappa_{2016} = 0.536$. These indices are a proxy for private construction costs. These historical land rents are residuals inferred from observed housing rents and construction costs. They differ from p_0, the land-sales-based charge used to anchor the fitted 2016 equilibrium. All are expressed as monthly flows.

The slope. Measuring cumulative development relative to 2006, set $Q_{2006} = 0$ and $Q_{2016} = \theta_H\tau$. Under a stable schedule, the land-supply curve $p = \bar{p} + bQ$ gives $b = (p_{2016} - p_{2006})/(\theta_H\tau)$. The theory writes $p = bQ$. The intercept can be absorbed into both per-unit-input costs, replacing κ_j by $\kappa_j + \bar{p}$. This is an equivalent parameterization: it preserves relative returns and the slope condition for opposite-signed rent responses. In the quantitative model we set $\bar{p} = p_0 - b\theta_H\tau = 81.27$, so that the fitted market builds τ at p_0.

Table A5 shows b under alternative assumptions. The calibrated value stays between 6.95 and 11.58 when the real rate, the cost index or the source of the 2016 land rent changes. It is 19.42 if the real construction cost stayed constant. Drawing the line through the origin and the 2016 point gives $b = 33.03$, because that treats all of the land rent as the result

Table A5: The land-price slope under other assumptions

Assumption	b	$b\theta_L(\theta_H - \theta_L)$
Baseline: data rents, 2% rate, ArchSD cost ratio	9.21	9,495
2016 land rent from the land sales	8.47	8,733
Real rate 1.5%	10.25	10,570
Real rate 2.5%	8.11	8,359
Real rate 3.0%	6.95	7,166
Cost ratio from the Arcadis index, 2007Q1 for 2006	11.58	11,943
Real construction cost constant	19.42	20,017
Through the origin (level, not change)	33.03	34,049

Notes: Each row changes one input of the calibration in the first row, on the sample of households with heads aged 20–60. The last row draws the land-supply line through the origin and the 2016 point.

of new construction. The thresholds in all rows lie below the baseline external calibration $\gamma = 50{,}095$. The constant-cost and through-origin thresholds exceed the age-20–60 census lower limit of 16,619.

C.4 External Subdivision Evidence

Three steps take the estimate of Leung, Choy and Chau (2022) to γ. Their 2017–2018 survey covers 4,145 apartments in older buildings in four Hong Kong districts, 1,896 of them subdivided, and their logit relates subdivision to the monthly rent gap from subdividing.

Step 1: slope. The logit coefficient on the rent gap is 1.230. We read it as applying to one sample standard deviation of the gap, HK$7,333.98, and convert October 2017 dollars to 2006 dollars with the deflator 0.784737. The article does not state the scaling, so this interpretation is a calibration assumption; a reading per HK$1,000 would give a different benchmark.

Step 2: average marginal effect. The average marginal effect, AME_{gap}, is the slope times the sample mean of $\pi_i(1-\pi_i)$, where π_i is apartment i's predicted probability of subdivision. The article does not report the π_i. For a logit fitted by unpenalized maximum likelihood, the log likelihood equals minus the sum of the binary entropies $H(\pi_i) = -\pi_i \log \pi_i - (1-\pi_i)\log(1-\pi_i)$, so the reported -919.158 fixes the mean entropy at 0.221751. One number fixes this average but not the distribution of the π_i, and the mean of $\pi_i(1-\pi_i)$ differs across the distributions

that share it. Using the entropy restriction alone, its lower bound is 0.054738, attained when predictions take the values approximately 0.058 or 0.942, and its upper bound is 0.079980, attained by mixtures of 0, 1/2 and 1; these bounds follow from Jensen's inequality and from $H(\pi) \geq 4\log(2)\,\pi(1-\pi)$. Hence AME_{gap} lies between 1.170×10^{-5} and 1.709×10^{-5} per 2006 monthly HK dollar. The baseline uses the midpoint, 1.440×10^{-5}.

Step 3: from the effect to γ. The model counterpart of the article's apartments, subdivided or not, is the landlords' existing stock counted in apartments of H size,

$$h_H^0 + \frac{\theta_L}{\theta_H} h_L^0 = d_H - \tau + \frac{\theta_L}{\theta_H} d_L = 0.283592,$$

where $d_L = 0.242384$ and $d_H = 0.226871$ are the fitted private demands for L and H. Conversion leaves this stock unchanged, so it does not depend on γ. Subdividing an H apartment yields θ_H/θ_L L dwellings, so its rent gap is $(\theta_H/\theta_L)[r_L - (\theta_L/\theta_H)r_H]$. A one-dollar rise in r_L therefore raises the gap by θ_H/θ_L dollars and the subdivided share of these apartments by $(\theta_H/\theta_L)\text{AME}_{\text{gap}}$, and each newly subdivided apartment adds θ_H/θ_L dwellings of quality L. Equating this response with $1/\gamma$, the response of landlord L supply in equation (9), gives

$$\frac{1}{\gamma} = \left(\frac{\theta_H}{\theta_L}\right)^2 \left(h_H^0 + \frac{\theta_L}{\theta_H} h_L^0\right) \text{AME}_{\text{gap}}, \tag{62}$$

so $\gamma = 42{,}190$ for the larger effect, 61,645 for the smaller and 50,095 at the midpoint. This step assumes that the response in the article's older-building sample carries over to the model's existing stock. These are conditional mapping bounds, not a confidence interval for γ. They omit the coefficient's sampling error, scaling uncertainty, and uncertainty about transferability to the modeled stock and the reverse conversion margin.

C.5 Conditional Stock-Accounting Scenarios

The calculations in this subsection show how historical stock changes map into a conversion parameter under additional accounting and behavioral assumptions. The bootstrap limits measure census sampling variation conditional on this mapping, excluding uncertainty about stock inflows and the common reference configuration.

Quantities. In each census we count private tenant households in each quality group, under the sample rules of Appendix C.1: income above HK$3,000 in 2006 and HK$5,578.78 in 2016 at 2006 prices. In 2006 quality is the rent itself, because 2006 is the base year; the census of that year has no floor area. We scale counts to households by the ratio of the official count of renting households to census tenant records in each year. Public quarters are types 1 and 11 in 2006 and 1 and 2 in 2016.

Net conversion. Let D be the net number of L dwellings created by conversion between the censuses, and X_H any growth in the H rental stock from outside, such as owner-occupied flats that come to the rental market. Stock accounting gives

$$S_{H,2016} - S_{H,2006} = T + X_H - \frac{\theta_L}{\theta_H} D.$$

Table A6 sets $X_H = 0$ and recovers D from this identity.

The behavioral mapping is a separate step. Define $w_t = r_{L,t} - (\theta_L/\theta_H) r_{H,t}$. The calculation uses

$$\gamma D/N = w_{2016} - w_{2006}.$$

This requires interpreting D/N as the change in conversion relative to a common reference stock, with the same marginal-cost schedule and normalization at both dates. The stock identity alone does not establish this interpretation. The negative level of the gap at both dates must be reconciled with that reference configuration; it does not prevent a positive change in the gap. A small D with a positive change in the gap means a high γ. Since D is close to zero, its upper limit gives a lower limit for γ. The change in the gap is the difference of two larger rent changes, so it depends on θ_L/θ_H: a rise of 0.01 in this ratio lowers the change by 43 a month and the lower limit on γ in proportion.

The sign condition and sensitivity to stock inflows

The condition. For positive D and a positive change in the rent gap, substituting the census-implied γ into $\gamma > b\theta_L(\theta_H - \theta_L)$ gives $(D/N)\, b\theta_L(\theta_H - \theta_L)/(w_{2016} - w_{2006}) < 1$, where w is the rent gap. We report this ratio because it stays finite when D is near zero. Its 95% upper limit is 0.57 among heads aged 20 to 60 and 0.21 among all tenants, from 2,000 bootstrap

draws of households within each census. The draws cover sampling error, and treat T and costs as known. Among heads aged 20 to 60, the point estimates give $\gamma = 36{,}992$, but that number depends on how T is assigned to this age group. Among all tenants T needs no age-group assignment, and the H stock grew by more than T. The resulting negative point estimate of D, combined with a positive change in the rent gap, does not yield a positive point estimate of γ under the maintained equation. The reported upper-limit calculation should therefore be distinguished from a feasible point calibration.

Other growth in the stock. A positive X_H raises inferred D and, where $D > 0$, lowers the implied γ. Setting the ratio to one gives the value of X_H at which the condition fails: 10,266 among heads aged 20 to 60 and 18,062 among all tenants, or 20% and 28% of the growth in private rental stock not explained by T. These thresholds hold the other inputs fixed. A joint change in the interpretation of development and other stock inflows would also require reconsidering the mapping into b. An exogenous addition to the L stock does not enter this H-stock accounting calculation or change the slope condition when the supply parameters are held fixed. An extra supply of L that responds to r_L changes the size of the rent movements, but the two rents still move in opposite directions exactly when $\gamma > b\theta_L(\theta_H - \theta_L)$.

C.6 Implementation of Public Allocation

In Hong Kong, eligible households queue by date of registration, and the income limit for eligibility rises with household size (Hong Kong Housing Authority, 2016). We summarize where the stock of flats ends up with the rule of Section 5. The targeted pool contains applicants of total mass μ, equal to public capacity when the household population is normalized to one. A share λ of public flats is allocated uniformly within this pool; the remaining flats are allocated by an even lottery among all applicants not already served. We rank applicants by equivalized income as a simple stand-in for limits that rise with household size, while private renters still choose between L and H by total income. Let $a = \mathbf{1}\{I < I_{HC}\}$ indicate an applicant, and let $t \in [0, a]$ be a household's probability of belonging to the targeted pool of poorest applicants. Thus the pool has mass μ, while targeted recipients have

Table A6: Conditional calibration from historical stock accounting

	Head aged 20–60	All ages
Panel A. Change between the censuses		
Private L dwellings, 2006 / 2016	125,941 / 180,971	145,922 / 208,583
Private H dwellings, 2006 / 2016	131,173 / 166,830	140,610 / 186,271
New rental dwellings, T	39,203	44,506
L rent, 2006 / 2016	3,000 / 5,324	3,000 / 5,336
H rent, 2006 / 2016	8,000 / 12,274	8,000 / 12,263
Land rent p, 2006 / 2016	83.79 / 115.21	83.16 / 114.09
Rent gap $r_L - (\theta_L/\theta_H)r_H$, 2006 / 2016	−618 / −227	−597 / −178
Panel B. Conditional mappings		
Land-price slope b	9.21 (0.23)	10.97 (0.27)
$b\theta_L(\theta_H - \theta_L)$	9,495	11,410
Net conversion to L, $(T - \Delta S_H)\theta_H/\theta_L$	7,839 (4,731)	−2,568 (5,316)
Conditional lower limit on γ (95%)	16,619	55,386
$b\theta_L(\theta_H - \theta_L)/\gamma$, upper limit (95%)	0.57	0.21
Panel C. Stock inflows at the sign-condition boundary		
Growth in private rental stock not from T	51,484	63,816
Of which into H at which the condition fails	10,266	18,062

Notes: Dwellings are private tenant households in each quality group, scaled to official renter counts. Rents and the land rent are monthly 2006 HK dollars; the land rent is per square metre, $p = r_H/\theta_H - \kappa$, with κ moved between the years by public tender price indices. Standard errors in parentheses and the 95% limits come from 2,000 bootstrap draws of households within each census and cover sampling error only. The bound on γ uses the upper limit on net conversion. Calculations set other inflows into H to zero and assume a common conversion reference stock; these limits do not cover uncertainty in either assumption. A negative residual means H stock growth exceeds T and, with the positive change in the rent wedge, does not yield a feasible positive point calibration of γ.

mass $\lambda\mu$. The probability of a public flat is

$$P_C = \lambda t + (a - \lambda t)\, P_{\text{rand}}, \tag{63}$$

where the lottery rate P_{rand} fills the flats. When policy changes, λ stays fixed and P_{rand} adjusts.

An applicant in the reserved pool receives a flat with probability $\lambda + (1-\lambda)P_{\text{rand}}$, whereas another applicant receives one with probability P_{rand}. Public occupancy across income groups therefore informs λ. Occupancy also depends on applications: some higher-income households

prefer private H, and their participation depends on δ through I_{HC}. The observed occupancy gradient consequently helps fit demand and targeting together. This rule summarizes the cross-sectional allocation of public flats; the counterfactual response of applications is supplied by preferences and the maintained allocation rule. The fit does not separately identify how incumbent tenants or waiting-list applicants respond to a policy change.

C.7 Calibration Constraints and Search

The search keeps a candidate only if public capacity clears and private-market accounting holds. The candidate must also satisfy the sorting restriction $\mu \leq F(I_{HL}) < F(I_{HC}) < 1$ on the total-income distribution F and the rent ordering $r_C < r_L < r_H$. Its r_L must lie below the lowest sample income, so that every assignment with positive probability has positive consumption. Entry into L construction must be unprofitable, $r_L \leq k_L + p_0\theta_L$, and private H demand must cover τ. In the fitted market and at every reform shown, every assignment with positive probability has positive consumption, so welfare covers the full sample.

We use multistart local searches, differential evolution and local refinement, and report the best feasible candidate without claiming a global optimum. Of 59 attempts, 33 reach within 0.1% of the best value.

As an allocation sensitivity check, we allow lottery odds to decline with income as well as reserving flats for the targeted pool. When we add a logistic slope to the lottery and raise λ from 0 to 0.2, the best slope falls from 0.67 to 0.08, the error rises only from 2.33% to 2.95%, and the share of flats held by would-be H renters stays at about 12%. This comparison supports the parsimonious single-parameter allocation rule for the baseline fit. Sampling uncertainty for the demand parameters is not estimated.

C.8 Recovering the Baseline Reference Stocks

Let d_L and d_H denote fitted private demands as shares of the normalized household population. The baseline conversion variable is $\Delta = [r_L - (\theta_L/\theta_H)r_H]/\gamma$, and new H construction is τ.

Table A7: 2016 data and model fit

Moment	Data	Model	Use
Public occupancy, poorest μ (%)	68.24	67.39	Target
Public occupancy, Q1 (%)	72.38	67.43	Target
Public occupancy, Q2 (%)	65.56	67.43	Target
Public occupancy, Q3 (%)	66.21	64.71	Target
Public occupancy, Q4 (%)	52.51	51.40	Target
Public occupancy, Q5 (%)	8.72	14.40	Not targeted[a]
Private-H share (%)	47.97	48.35	Target
Low-quality rent	5,324.04	5,311.71	Target
High-quality rent	12,273.92	12,111.36	Target[b]

Notes: Quintiles are equivalized-income quintiles of the sample households. Rents are monthly 2006 HK dollars. The private-H share is conditional on private tenure. [a]Fixed capacity makes Q5 occupancy equal to 5μ minus Q1–Q4 occupancy, so it is not an independent check. [b]The H-rent term enters the objective, but its model value is fixed by the cost inputs.

Stock accounting then recovers

$$h_L^0 = d_L - \Delta, \qquad h_H^0 = d_H + \frac{\theta_L}{\theta_H}\Delta - \tau. \tag{64}$$

The conversion rent gap, $r_L - (\theta_L/\theta_H)r_H$, is -165.43. Thus $N\Delta$ equals $-2{,}448$ L units, and $(h_L^0, h_H^0) = (0.245686, 0.172485)$. These are model reference stocks recovered from the fitted equilibrium, not independently observed historical stocks. The baseline level Δ is distinct from the between-census conversion change D/N in Appendix C.5.

C.9 Uniqueness of the Equilibrium

Corollary 1 gives existence and uniqueness for every $\lambda \in [0, 1]$ under the rule of Section 5, and its proof extends to ranking by equivalized income. Each sample household is a segment along which income and rank rise together, so all fractions come from one joint distribution whose income marginal F is continuous. Adding applicants can only lower an existing applicant's probability of being in the reserved pool, and for $\lambda < 1$ it also lowers the lottery rate, so that applicant's probability of a flat weakly falls. Private L demand therefore rises weakly with both cutoffs. The number of flats is fixed, so a change in the cutoffs moves demand by at most the mass of households between the old and new cutoffs. Demand is thus continuous,

including at $nF_C = \mu$, where it is zero. Counting gives

$$(nF_L - \mu)_+ \leq D_L \leq \min\{nF_L,\, nF_C - \mu\},$$

because at most μ of the nF_L households who prefer L get a flat, and all μ flats go to applicants.

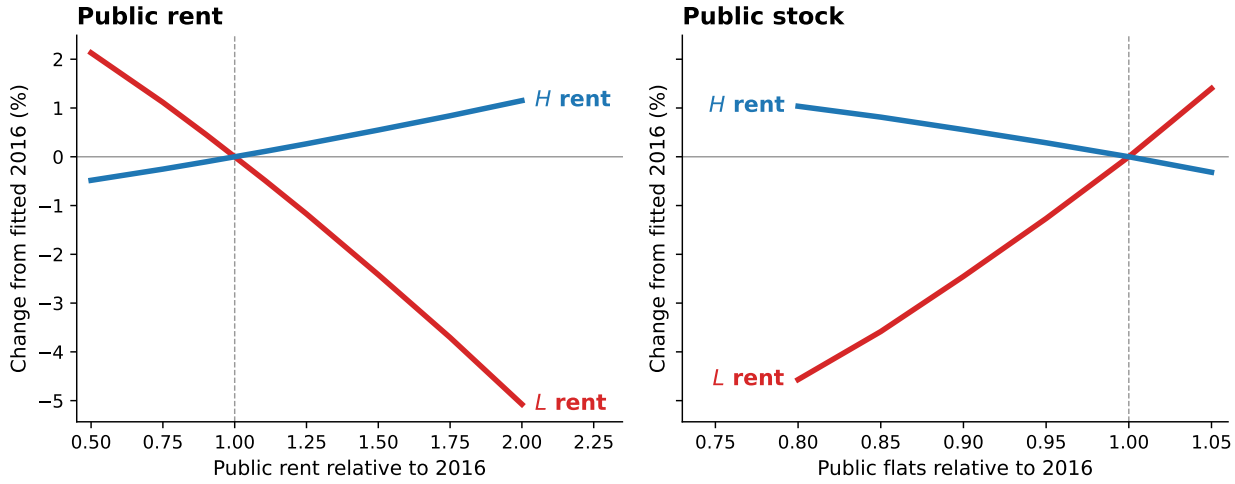
These properties give the same unique root under conditions (NM1) and (NM2). At the root, $0 < x^* = D_L \leq nF_L$ gives $I_{HL} > I_{\min}$, and $x^* < \bar{x} < X$ gives positive construction and both private qualities. Condition (NM1) excludes an equilibrium with only H. An equilibrium without construction or with only L has r_H below $R_H(\bar{x})$, so $D_L \leq nF(I_{HC}(\bar{x})) - \mu < \bar{x}$ by condition (NM2), which is less than the L stock such an equilibrium needs. This argument does not require $F_C < 1$, so it covers bounded income support. With b calibrated, the land-price intercept $\bar{p}$ is part of κ_L and κ_H in these conditions.

The conditions of the corollary hold at the baseline and at every point of Figure A8, with $n = 1$ and x the private L share. At the baseline, condition (NM1) holds with $F(I_{HL}(x_\ell)) = 0.861$ against $\mu + x_\ell = 0.674$. Condition (NM2) holds at $\bar{x} = 0.27$, where $F(I_{HC}(\bar{x})) - \mu = 0.268$. On A, the supporting L rent is at most 5,446, below $I_{\min} = 5{,}582$. The solver searches all four construction regimes at each point and finds one equilibrium.

C.10 Policy Implementation and Adjustment

The experiments compare static equilibria after households and suppliers have adjusted. Their longer-run interpretation is conditional on the inherited stock and conversion technology; it is not a dynamic steady-state calculation. The model does not specify the path or duration of adjustment, or how incumbent tenure rights would affect that path. Applicants waited an average of 4.7 years at the end of 2016 (Legislative Council, 2017), and about 12,000 flats were recovered from departing tenants in the year to March 2017 (Transport and Housing Bureau, 2018). Reversing either policy change reverses the rent responses over the range reported in Tables A9 and A10.

Figure A8: Private rents under the two reforms



Notes: Each point solves the market again at the baseline parameters. The horizontal axes are relative to the 2016 public rent and stock. Developers build only H throughout.

C.11 Welfare Measurement

We measure welfare by expected utility per equivalent adult, $U_j(I, s)$ in equation (3), taken before the lottery. For a group of households $\mathcal{G}$, the person-weighted mean is

$$\overline{U}_{\mathcal{G}} = \frac{\sum_{i \in \mathcal{G}} n_i \left[P_{C,i}\, U_{i,C} + (1 - P_{C,i})\, U_{i,j(i)} \right]}{\sum_{i \in \mathcal{G}} n_i}, \tag{65}$$

where n_i is the number of family members in household i and $j(i)$ is the quality it rents if it does not get a public flat. Quintiles contain equal numbers of family members ranked by equivalized household income. Table A8 reports the percentage change in each group's mean. Its rent component applies counterfactual rents while holding baseline tenure assignments fixed; the allocation component is the remainder of the total change. The tenure-specific columns separate the rent component into public, private L and private H effects. Panel C of Tables A9 and A10 instead uses household quintiles and household weights.

C.12 Additional Results

Tables A9 and A10 report allocation, private-market adjustment and household-weighted welfare separately for each reform. Table A11 reports the calibration and fit of alternative

Table A8: Change in expected utility by income group, and its three channels

Group	Net	Who gets a flat	Rent changes	Rent changes, by tenure: Public	Private L	Private H
Panel A. Public rent 50% higher						
Q1	−3.7	0.3	−4.0	−5.9	2.3	—
Q2	−2.5	0.2	−2.6	−4.0	1.2	−0.2
Q3	−1.6	0.1	−1.7	−2.8	0.7	−0.2
Q4	−0.9	0.1	−1.0	−2.0	0.6	−0.2
Q5	−0.2	0.0	−0.2	−1.7	0.5	−0.1
All	−1.0	0.1	−1.1	−3.2	1.0	−0.1
Panel B. A tenth of public flats turned private						
Q1	−1.9	−2.5	0.5	—	2.3	—
Q2	−1.3	−1.6	0.3	—	1.2	−0.2
Q3	−1.2	−1.4	0.2	—	0.8	−0.2
Q4	−0.6	−0.6	0.0	—	0.6	−0.2
Q5	−0.1	−0.1	−0.1	—	0.5	−0.1
All	−0.6	−0.7	0.1	—	1.0	−0.1

Notes: Percentage change in the group's mean expected utility per equivalent adult, as in equation (65), measured before the lottery. Net is the full change. Rent changes holds every household in its original tenure and applies the new rents, and Who gets a flat is the remainder, the effect of the change in who holds a public flat. The last three columns split the rent changes: the percentage change in utility of persons who stay in each tenure. Quintiles split the 103,085 family members into equal fifths by equivalized income. Rents paid to the government are not returned to households.

specifications; Tables A12 and A13 report the corresponding policy outcomes.

In the specification with quintile odds, each income quintile has its own acceptance rate. Four parameters give the fall in log odds from Q1 to each other quintile, and a common level fills the flats. The fitted values are 0.32, 0.27, 0.55 and 1.62, and the fit has a root mean squared relative error of 0.47%. Acceptance falls sharply only in Q5, which is the part of the occupancy profile that the reserved share misses.

Table A9: Public-rent changes: allocation, private supply and welfare

	Baseline	r_C of 2006	$r_C + 50\%$	$2r_C$
Panel A. Policy and public allocation				
Public rent r_C	1,299	1,436	1,948	2,597
Public flats (% of households)	53.1	53.1	53.1	53.1
Applicants (% of households)	81.5	81.3	80.4	79.2
Public flats held by would-be H renters (%)	12.4	12.1	10.5	8.3
Lottery rate (%)	60.8	61.0	61.8	62.8
Acceptance in the reserved pool (%)	67.4	67.6	68.2	69.1
Panel B. Private-market adjustment				
L rent	5,312	5,286	5,183	5,042
H rent	12,111	12,125	12,178	12,251
Change in L rent (%)	0.0	−0.5	−2.4	−5.1
Change in H rent (%)	0.0	0.1	0.5	1.2
New H dwellings	39,203	39,461	40,487	41,898
Landlord net conversion to L	−2,448	−2,920	−4,792	−7,367
Land rent p	112.7	112.9	113.7	114.9
Panel C. Household-weighted welfare (%)				
Q1	0.0	—	−3.9	−8.1
Q2	0.0	—	−2.8	−5.6
Q3	0.0	—	−1.8	−3.7
Q4	0.0	—	−1.0	−2.1
Q5	0.0	—	−0.2	−0.5
All	0.0	—	−1.1	−2.3

Notes: Rents are monthly 2006 HK dollars; land rent is per square metre. Dwellings are counts in the sample market. Each column solves the equilibrium at $\gamma = 50{,}095$ and $b = 9.21$. Landlord net conversion is measured relative to the model reference stock and is negative when L dwellings are converted into H. Panel C reports percentage changes in mean expected utility per equivalent adult, using household quintiles ranked by equivalized income and equal household weights. Baseline changes are zero. Only public rent changes; public capacity is fixed. Welfare for the 2006-public-rent experiment is not reported (—).

Table A10: Conversion to private rental: allocation, private supply and welfare

	Baseline	10% conversion	20% conversion
Panel A. Policy and public allocation			
Public rent r_C	1,299	1,299	1,299
Public flats (% of households)	53.1	47.8	42.5
Applicants (% of households)	81.5	81.7	81.8
Public flats held by would-be H renters (%)	12.4	11.6	10.9
Lottery rate (%)	60.8	53.9	47.3
Acceptance in the reserved pool (%)	67.4	61.7	56.2
Panel B. Private-market adjustment			
L rent	5,312	5,181	5,069
H rent	12,111	12,179	12,237
Change in L rent (%)	0.0	−2.5	−4.6
Change in H rent (%)	0.0	0.6	1.0
New H dwellings	39,203	40,505	41,629
Landlord net conversion to L	−2,448	−4,826	−6,878
Land rent p	112.7	113.7	114.6
Panel C. Household-weighted welfare (%)			
Q1	0.0	−2.1	−4.0
Q2	0.0	−1.4	−2.7
Q3	0.0	−1.4	−2.9
Q4	0.0	−0.7	−1.3
Q5	0.0	−0.2	−0.3
All	0.0	−0.7	−1.4

Notes: Rents are monthly 2006 HK dollars; land rent is per square metre. Dwellings are counts in the sample market. Each column solves the equilibrium at $\gamma = 50{,}095$ and $b = 9.21$. Landlord net conversion is measured relative to the model reference stock and is negative when L dwellings are converted into H. Panel C reports percentage changes in mean expected utility per equivalent adult, using household quintiles ranked by equivalized income and equal household weights. Baseline changes are zero. The reform transfers the stated share of public flats into private L rental supply, holding public rent fixed. This policy transfer is distinct from landlord conversion between qualities.

Table A11: Calibration and fit across sensitivity specifications

Specification	b	$b\theta_L(\theta_H - \theta_L)$	γ	RMS error (%)
Baseline	9.21	9,495	50,095	2.92
γ low	9.21	9,495	42,190	2.92
γ high	9.21	9,495	61,645	2.92
b through origin	33.03	34,049	50,095	2.92
T by households	11.20	11,549	48,486	2.92
All ages	10.97	11,410	60,801	3.16
Six moments	9.21	9,495	50,124	1.37
Quintile odds	9.21	9,495	50,268	0.47
2.5% rate	8.11	8,359	50,192	6.85
3% rate	6.95	7,166	50,237	13.37
10% band	9.21	9,495	50,114	2.94

Notes: Each specification changes the indicated input or fitting choice from the baseline. γ low and high use the external calibration bounds. b through origin sets $b = p_0/(\theta_H\tau)$. T by households allocates construction by household share rather than private-tenant share. All ages drops the age restriction. Six moments drops Q1 and Q2 occupancy. Quintile odds allows acceptance odds to differ across income quintiles. Rate specifications change the real rate used for construction costs and land and refit the model. The 10% band spreads each reported income over 10% on either side rather than 5%.

Table A12: Sensitivity: public rent 50% higher

Specification	L rent	H rent	Q1 utility	Overall utility
Baseline	−2.4	0.5	−3.7	−1.0
γ low	−2.0	0.6	−3.8	−1.1
γ high	−2.9	0.5	−3.6	−1.0
b through origin	−0.6	1.2	−4.2	−1.2
T by households	−2.1	0.6	−3.8	−1.1
All ages	−3.2	0.7	−4.1	−1.3
Six moments	−2.5	0.6	−3.7	−1.0
Quintile odds	−1.8	0.4	−4.3	−1.1
2.5% rate	−2.4	0.4	−3.4	−1.0
3% rate	−2.4	0.3	−3.2	−1.0
10% band	−2.4	0.5	−3.7	−1.1

Notes: Entries are percentage changes relative to each specification's fitted baseline. Q1 and overall welfare use person-weighted mean expected utility per equivalent adult, with quintiles ranked by equivalized income. Table A11 defines the specifications and reports their parameter values and fit. Fiscal proceeds are not redistributed.

Table A13: Sensitivity: 10% conversion to private rental

Specification	L rent	H rent	Q1 utility	Overall utility
Baseline	−2.5	0.6	−1.9	−0.6
γ low	−2.1	0.6	−2.0	−0.7
γ high	−2.9	0.5	−1.8	−0.6
b through origin	−0.6	1.2	−2.5	−0.8
T by households	−2.1	0.6	−2.0	−0.7
All ages	−3.1	0.7	−2.5	−0.8
Six moments	−2.5	0.6	−2.0	−0.6
Quintile odds	−2.2	0.5	−2.2	−0.7
2.5% rate	−2.1	0.3	−2.1	−0.6
3% rate	−1.8	0.2	−2.1	−0.6
10% band	−2.4	0.5	−2.0	−0.6

Notes: Entries are percentage changes relative to each specification's fitted baseline. Q1 and overall welfare use person-weighted mean expected utility per equivalent adult, with quintiles ranked by equivalized income. Table A11 defines the specifications and reports their parameter values and fit. Fiscal proceeds are not redistributed.