Intermediation and Relational Contracts in Competitive Markets

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July 2024

Abstract

We study intermediation in frictionless matching markets for relational contracts. We show that intermediaries that aggregate demand, monitor performance, and direct allocations can efficiently emerge. Such intermediaries reduce producer idleness, separation, and incentive rents, but require an additional intermediary incentive rent. Buyers sort into bilateral and intermediated relational contracts in a unique steady-state equilibrium. Their optimal choice depends on demand volatility, gains from specialization, market tightness, the extent of the market, and reputation concerns. A decline in communication frictions may increase intermediation. Our model can be used to understand the labor boundaries of firms and the welfare consequences of intermediation.

Keywords: intermediaries, relational contracts
JEL: D23, L22, L24

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1 Introduction

The choice between bilateral and intermediated exchange relationships pervades the economy. Many examples can be found in global value chains. For instance, Kenyan flower exporters can either sell directly through bilateral *relational contracts*—collaborations sustained by the value of future relationships—or sell in a spot market operated by a trusted intermediary (Macchiavello and Morjaria 2015). Retailers like Walmart can either source consumer goods directly from manufacturers or procure through global sourcing firms, such as Li & Fung (Belavina and Girotra 2012). These decisions influence the structure and efficiency of buyer-supplier networks. They also affect the distribution of rents across firms and nations.

Labor markets are also riddled with the choice between bilateral and intermediated contracts. To obtain professional services, such as cleaning, security, accounting, HR, or legal services, entrepreneurs can either employ workers or engage an intermediary firm that flexibly allocates workers to clients (Abraham and Taylor 1996). With the rise of the gig economy, Internet users can either form direct relationships with service providers or contract through online platforms, such as Airbnb, Upwork, or Uber. These choices shape the labor boundaries of firms and affect the wages, employment, and skill investment of workers. They also influence the efficiency of labor markets.

Given their ubiquity, it would be useful to have equilibrium frameworks for analyzing these choices between bilateral relational contracts and intermediated multilateral trade. Sadly, existing models are largely silent regarding such decisions. Neoclassical theory assumes the existence of centralized markets where goods trade without intermediaries. Recent models of intermediation focus on search frictions and adverse selection, ignoring the possibility that trade may be subject to opportunism and hence require relational contracts.[1] Even though vast literatures have underscored the prevalence of intermediaries[2] and the importance of trust and relationships in

[1]See, e.g., Antras and Costinot (2011); Glode and Opp (2016); Nosal, Wong and Wright (2015, 2019); Biglaiser and Li (2018); Rhodes, Watanabe and Zhou (2021).

[2]According to Wallis and North (1986), the private "transaction sector" was around 41% of U.S. GNP in 1970. Spulber (1996) estimates that intermediaries account for about 25% of U.S. GDP. Ahn, Khandelwal and Wei (2011) show that intermediaries account for around 20% of China's exports in 2005. Berlingieri (2013) documents that professional and business services alone account for 12% of US GDP. Bernard, Grazzi and Tomasi (2015) document that more than one-quarter of Italian exporters are intermediaries, and they account for over 10% of exports. See

exchange,[3] there is little formal theory that explores how exchange is shaped by intermediaries that enter relational contracts with economic agents on both sides of a market.

In this paper, we put forth a theory of intermediation in markets for relational contracts. Specifically, we analyze intermediaries that track the performance of multiple producers, direct them to serve buyers as demand fluctuates, and punish poor performance with contract termination. Our analysis addresses several questions: Under what conditions can such intermediaries profitably emerge? Why would some buyers engage intermediaries while others bilaterally contract with producers? How does intermediation alter the terms and patterns of trade? How does a reduction in communication friction, such as the advent of the Internet, affect the role of intermediaries? In answering these questions, we aim to shed light on the determinants and consequences of intermediation choices in a variety of empirical settings.

Section 2 develops our proposed framework. We consider a simple repeated-game model of an exchange economy. In this economy, written legal contracts are insufficient to compel producers to perform, so buyers must motivate producers using future surplus in long-term contractual relationships. They meet and enter contracts in frictionless matching markets, as modeled by Shapiro and Stiglitz (1984) and MacLeod and Malcomson (1989, 1998). We differ from these models by assuming that buyer demand may fluctuate. Buyers with fluctuating needs have less expected future surplus to promise and cannot easily sustain bilateral contracts.

We show in this environment that intermediaries can improve efficiency by aggregating the fluctuating demand of buyers. The key assumption needed for this result is that intermediaries can observe the performance of multiple producers. Intermediaries can therefore direct producers to service different buyers as demand fluctuates and track the performance of producers as they move across buyers. Since this prevents producers from becoming idle or unmatched, intermediaries can offer lower incentive rent and still induce producers to perform.

The cost of intermediation, however, is that the buyer must pay a marked-up service fee to incentivize the intermediary to perform its duties. In other words, intermediation results in a

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also books by Spulber (1999) and Krakovsky (2015).

[3] See, e.g., Geertz (1962, 1978); Macaulay (1963); Macneil (1978); Granovetter (1985); Powell (1990); Bernstein (1992, 2016); Dyer and Singh (1998); Fafchamps (2004); Greif (2006); Gibbons and Henderson (2012).

form of double marginalization.[4] For this reason, buyers with sufficiently persistent demand prefer to directly contract. Bilateral contracts are also preferred if the producer market is slack and the opportunity cost of keeping a producer idle is therefore low.

Our two main propositions show that buyers sort into bilateral and intermediated contracts in a unique steady-state equilibrium. In equilibrium, if producer idleness is sufficiently costly, centralized intermediaries emerge as a contracting nexus between producers and the subset of buyers whose business needs fluctuate. The remaining buyers contract with producers in a sea of decentralized pairings.

We establish our main propositions by comparing bilateral and intermediated contracts in Section 3. Section 4 characterizes the steady-state equilibrium. We then derive several additional implications regarding the determinants and consequences of intermediation. First, directly contracted producers have higher average pay, more dispersed pay, higher separation rates, and higher idleness in equilibrium than intermediated producers. Second, when intermediaries are introduced into an economy where producers may freely enter, buyers with fluctuating demand benefit, while producers initially earning pay premiums experience pay reductions. Third, intermediation is profitable only if there is a sufficiently large market of buyers and producers.

Section 5 extends the model to analyze the relationship between intermediation, specialization, and reputation concerns. We show that intermediated producers are more specialized and that intermediation increases producer specialization. Moreover, buyers are more likely to choose intermediation if there are gains from producer specialization and if the intermediaries have reputation concerns. In contrast to models of intermediaries based on search frictions and asymmetric information, we find that improvements in communication technologies may *increase* intermediation. The reason is that ease of communication may allow buyers to better coordinate on collective punishments against intermediaries who shirk. This lowers intermediary service

[4]In the classic literature in industrial organization, double marginalization occurs when both upstream and downstream parties possess market power and use restrictive linear contracts. This form of double marginalization can be eliminated by vertical integration of pricing decision rights, resale-price maintenance, or non-linear pricing (Tirole 1988). In our model, double marginalization instead arises from the non-verifiability of contract performance and the non-transferability of production and intermediation capabilities, like models of middleman margins (Biglaiser and Friedman 1994; Bardhan, Mookherjee and Tsumagari 2013). Conventional remedies do not eliminate this form of double marginalization.

Table 1: Examples of situations where bilateral and intermediated contracts may coexist

Intermediaries	Buyers	Producers
Professional service firms (e.g., law, accounting, HR, cleaning, security, consulting)	Clients	Workers
Retailers, wholesalers, and e-commerce platforms (e.g., Walmart, Amazon, Alibaba, eBay, Etsy)	Customers	Sellers
Franchisors (e.g., Marriott, Starbucks, McDonald's, UPS)	Consumers	Franchisees
Ride-sharing platforms (e.g., Uber, Lyft, Grab, Didi)	Riders	Drivers
Online rental markets (e.g., Airbnb, Turo)	Renters	Owners
Online labor markets (e.g., Upwork)	Businesses	Freelancers
Hospitals and clinics	Patients	Doctors
Schools and universities	Students	Teachers

fees, so intermediation becomes more attractive.

Section 6 discusses real-world applications. In an application of particular importance, we study the determinants and effects of professional service outsourcing. We show that the model provides a plausible framework for understanding recent empirical findings. As such, our model sheds new light on classic questions concerning the boundaries of the firm that were first raised by Coase (1937). Our model can also used to understand the structure of supply chains, franchises, online platforms, and many other intermediary organizations. Table 1 provides a list.

Our work contributes to a growing literature that embeds relational contracts in markets and derives aggregate implications (Shapiro and Stiglitz 1984; MacLeod and Malcomson 1989, 1998; Yang 2008; Board and Meyer-ter Vehn 2015; Fahn 2017; Powell 2019; Fahn and Murooka 2022; Li 2022).[5] The novelty of our contribution is to analyze intermediation in frictionless markets for relational contracts. We show that by aggregating fluctuating demand, intermediaries can ease the tension between dynamic allocations and incentive provision, and thereby enable more trade and specialization. In related work, Board (2011) shows that the tension between incentive

[5]For surveys of recent work on relational contracts, see MacLeod (2007), Malcomson (2013), Gil and Zanarone (2017), and Macchiavello and Morjaria (2023).

provision and dynamic allocations can lead buyers to restrict their number of trading partners. Andrews and Barron (2016) show that the optimal dynamic allocation among multiple producers depends on payoff-irrelevant past performance.[6] Li and Powell (2020) show that interacting in multiple activities can strengthen collaborations in the presence of random fluctuations due to an aggregation mechanism similar to ours.[7]

Our work also advances the literature on intermediaries. One strand of this literature studies certification intermediaries who overcome adverse selection (Biglaiser 1993; Lizzeri 1999; Glode and Opp 2016; Biglaiser and Li 2018). Another studies middlemen who overcome search frictions (Rubinstein and Wolinsky 1987; Gehrig 1993; Yavaş 1994, 1996; Rust and Hall 2003; Antras and Costinot 2011; Wright and Wong 2014; Nosal, Wong and Wright 2015, 2019; Rhodes, Watanabe and Zhou 2021). Very few papers examine how intermediaries may overcome moral hazard. Most relatedly, Biglaiser and Friedman (1994) provide a model in which middlemen with reputations have a larger incentive to monitor than an ordinary buyer does, and that they are in a better position to learn about quality than a typical consumer because they buy a larger proportion of the producers' goods.[8] In contrast, middlemen do not have reputations in our baseline model. They improve performance simply by entering multiple relational contracts on both sides of the market, monitoring performance, and directing exchange as buyer demand fluctuates. In the supply chain management literature, Belavina and Girotra (2012) show a similar mechanism using a less tractable model with two buyers, two suppliers, and non-transferable utilities.

Finally, our work is related to a literature on market institutions (e.g., North 1981; Landa 1994; Greif 1993, 2006; Fafchamps 2004; Calvert 1995). For example, Milgrom, North and Weingast (1990) show that private judges in medieval trade can create trust with less observability than reputation systems. Greif, Milgrom and Weingast (1994) highlight the role of merchant guilds who mechanically announce boycotts of a city who cheats its traders. Ramey and Watson (2002)

[6]See also Tunca and Zenios (2006), who show that bilateral relational contracts may dominate the use of a procurement auction.

[7]Relatedly, Fong and Li (2017) show that relational contracts can be deepened by the presence of a non-strategic supervisor carrying out subjective performance reviews. Troya-Martinez and Wren-Lewis (2023) explore a model where a manager may receive kick-backs and show that such managers can improve relational contracts in environments with commitment difficulty.

[8]Bardhan, Mookherjee and Tsumagari (2013) embed this model of middlemen in a general equilibrium theory of occupational choice to study the effects of trade liberalization.

highlight the usefulness of a non-strategic intermediary that can verify past actions.[9] Relative to this literature, we make several contributions. First, we highlight a novel trade-off between the benefit of intermediation due to the aggregation of short-term interactions and its cost in the form of double marginalization. Second, we derive novel and sharp predictions regarding the determinants and welfare consequences of relational intermediation in a large competitive market. Third, we show that a model of relational intermediation can explain a variety of recent empirical findings regarding the labor boundaries of firms. This final contribution not only advances the literature on the theory of the firm, but also highlights the potential for private-sector firms to complement legal and regulatory institutions in enabling trust and exchange.

2 Model

Basics. Time is discrete and infinite, $t \in \{0, 1, \ldots\}$. There are a unit mass of buyers indexed by i who demand services. There is a continuum of producers indexed by j who provide services. The measure of producers is determined by endogenous entry at the start of each period subject to entry cost C, which represents the producers' training or opportunity cost. There is a finite number K of intermediaries indexed by k who neither directly demand nor provide services. All players are infinitely-lived and have a common discount factor $\delta \in (0, 1)$.

Demand realization. After the endogenous entry of producers, the service demand of each buyer i, denoted $d_{it} \in \{0, 1\}$, is realized and publicly observed.[10] Demand d_{it} is redrawn at the beginning of each period t following a Markov process. The demand-switching probabilities are each buyer's publicly known type $\alpha_i = (\alpha_{1i}, \alpha_{0i})$. With probability α_{1i}, buyer i's demand switches from 1 to 0. With probability α_{0i}, buyer i's demand switches back from 0 to 1. The distribution of buyers types is a distribution F on $[0, 1] \times [0, 1]$.

[9]More recently, Budish (2023) argues that blockchain-based trust systems must incentivize validators against attacking the system and therefore is an expensive trust technology without economies of scale.

[10]To focus on the relational incentives, we abstract from any adverse selection problem in the model. Therefore, when contracting with buyers, producers know whom they are dealing with and have the correct expectation of how the relationships will go.

Matching. Both buyers and intermediaries can offer contracts to producers in a producer market. In addition, buyers can offer contracts to intermediaries in an intermediary market. Each contract is a contingency plan specifying compensation, effort levels, and the probability of continuation when demand switches. Following Board and Meyer-ter Vehn (2015), matching is frictionless, meaning that all offers are accepted subject to participation constraints.

In the producer market, matching is random and anonymous in that all unmatched producers have the same probability of being matched with a given buyer or intermediary regardless of their histories; matched producers do not receive contract offers.[11] In the intermediary market, matching is not anonymous, so matching probabilities may depend on the observable history of an intermediary. Since intermediaries cannot produce by themselves, they meet the requirements of contracts with buyers by entering contracts with producers.

We assume that an intermediary can match with a continuum of buyers and producers. However, in any period, buyers can match with at most one producer or intermediary, while producers can match with at most one buyer or intermediary.[12] Matching in intermediary market precedes matching in the producer market, so intermediaries can always fulfill the service demands of their matched buyers if there is an excess of producers.

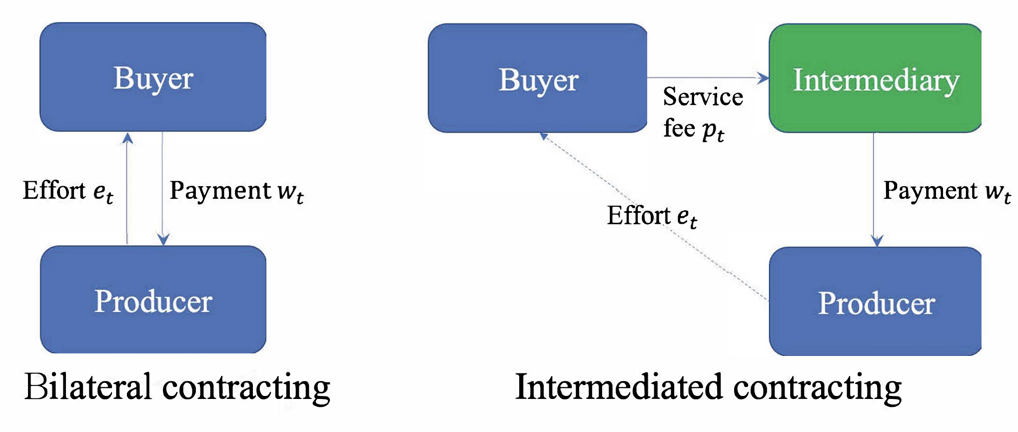
Bilateral contracts. If a buyer matches with a producer, the remainder of the period proceeds as follows. First, the buyer makes a payment $w_t \geq 0$ to the producer.[13] The producer then chooses an effort level, denoted by $e_t \in \{0, 1\}$. The cost of effort is given by $c(e_t)$, where $c(0) = 0$ and $c(1) = c > 0$. The effort generates an output for the buyer only if demand is positive, so $y_{it} = y d_{it} e_t$. Effort and output are observable by the buyer but are not verifiable by a court. We

[11] The assumption of random and anonymous matching simplifies analysis but is not important. Similar results can be obtained under the assumption that buyers and intermediaries first offer contracts to unmatched producers with whom they had previously matched. Relatedly, Board and Meyer-ter Vehn (2015) analyzes relational contracts in a frictionless matching market where matched producers may receive on-the-job offers. They show this leads to heterogeneous productivity across otherwise identical firms. We rule out this possibility for simplicity.

[12] This assumption rules out the possibility that producers and buyers become intermediaries themselves.

[13] Here we do not allow for ex-post bonus payments, which play in important role in MacLeod and Malcomson (1998) and Levin (2003). This assumption is not important: There is an excess of producers, so buyers in our model have all the bargaining power. It can then be shown that for any contract with bonuses, there is a weakly more profitable contract for buyers without bonuses. Intuitively, the buyer prefers to pay the producer for effort e_{t-1} at the latest possible moment before e_t. It is therefore weakly profitable to shift bonus b_{t1} into next period's pay w_t. See Board and Meyer-ter Vehn (2015).

Figure 1: Illustration of bilateral and intermediated contracts



assume that $y > \frac{c}{\delta} + (1-\delta)C$, so that for buyers with unchanging and positive demand, there is always enough surplus to incentivize producer effort.

Intermediated contracts. If a buyer matches with an intermediary, she pays a service fee $p_t \geq 0$ to the intermediary. The intermediary chooses to assign one or none of its producers to the buyer. The intermediary pays w_t to the producer. The producer then exerts costly effort e_t and produces output y_{it} for the buyer.[14] Effort and output are observable by both the buyer and the intermediary, but are not verifiable by a court.

Separation. Either party in a match can choose to terminate their contract and separate from each other both after demand realization and after production. If separation occurs after demand realization, both parties can participate in the producer or intermediary market matching in the current period. However, if separation occurs after production, they need to wait till the next period to rematch.

Payoffs. In a bilateral contract, the buyer's payoff is $y_{it} - w_t$ in each period t. In an intermediated contract, the buyer's payoff is $y_{it} - p_t$. The intermediary's payoff per service demand is $p_t - w_t$. The producer's payoff is $w_t - c(e_t)$.

[14]Similarly, we do not allow for ex-post service fees. This is unimportant for the same reason that ex-post bonuses are assumed away; buyers can immediately rematch with new intermediaries, and contracts with ex-post service fees are weakly less profitable for buyers.

Remark. This model has two key features that are not present in standard models of relational contracting in frictionless matching markets, such as Shapiro and Stiglitz (1984). First, we allow buyers to have fluctuating demand. Second, we introduce intermediaries who can enter relational contracts with a multitude of agents on both sides of the market. These intermediaries do not have intrinsic demand or productive ability. What they can do, however, is to match with a large set of buyers and producers and monitor the performance of all of their matched producers. As we shall show, when demand is volatile and the cost of idleness is high, intermediaries can sustain cheaper relational contracts on both sides of the market by reassigning their producers across buyers. Buyers may therefore prefer intermediation rather than bilateral contracting.

3 Bilateral and Intermediated Relational Contracts

In this section we characterize optimal bilateral and intermediated relational contracts. We then analyze the buyer's choice between bilateral and intermediated contracts. This analysis explains why and when intermediation may be preferred over bilateral contracting.

3.1 Bilateral Relational Contracts

To analyze bilateral relational contracts, we take the perspective of a single buyer i. Since producers are homogeneous, we omit the j subscript. We assume that the producer's pre-matching continuation value is exogenously given as $\overline{U} > 0$. We endogenize $\overline{U}$ in Section 4.

We say that strategies under a relational contract are *contract-specific* if they do not depend on the player's identity, calendar time, or any history outside the current contract. A contract is *stationary* if strategies are time-invariant functions of the buyer's demand realizations. A contract is offerer-optimal if it yields the highest possible surplus for the party offering the contract. We restrict our attention to buyer-optimal, contract-specific, stationary contracts in which producer's effort level is one if and only if $d_{it} = 1$.[15] These contracts must also satisfy the following two

[15]Focusing on stationary relational contracts is without loss when imposing pairwise stability (or "bilateral efficiency" in Board and Meyer-ter Vehn (2015)) on the equilibrium concept. A pairwise stable relational contract is a Pareto-optimal contract for parties in a match when they take their outside options as given. See Li (2022) for a more detailed discussion on how non-stationary relational contracts may be optimal in equilibrium when the

conditions. First, on the equilibrium path, parties within a match always choose to continue their relationship immediately after production. Second, off the equilibrium path, deviations are punished in the harshest possible way. These assumptions are standard in the literature (MacLeod and Malcomson, 1998; Baker, Gibbons and Murphy, 2002; Board and Meyer-ter Vehn, 2015).

Let $C_i^B = (w_{1i}, w_{0i}, \beta_i)$ denote a contract-specific, stationary relational contract offered by buyer i directly to a producer. In this contract, w_{1i} is the payment when $d_{it} = 1$, w_{0i} is the payment when $d_{it} = 0$, and $\beta_i \in [0, 1]$ is the probability that buyer i stays with the producer when the buyer's demand switches from one to zero. The time subscript is dropped since we focus on stationary contracts.

Under a bilateral contract, the buyer motivates the producer to exert effort using credible promises of future surplus from their contractual relationship. If the buyer deviates from the specified payment, the producer exerts no effort and separates from the buyer after production with probability one. If the producer deviates from the specified effort, the buyer separates from the producer after production with probability one. Since the buyer has fluctuating demand, the retention probability β_i determines the expected duration of the relationship and therefore affects the level of payments needed to incentive the producer.

If $\beta_i > 0$, the post-matching continuation payoffs for the producer when $d_{it} = 1$ and $d_{it} = 0$ are, respectively, given by

$$U_{1i} = w_{1i} - c + \delta\Big[(1 - \alpha_{1i})U_{1i} + \alpha_{1i}(\beta_i U_{0i} + (1 - \beta_i)\overline{U})\Big], \tag{1}$$

and

$$U_{0i} = w_{0i} + \delta\Big[\alpha_{0i}U_{1i} + (1 - \alpha_{0i})(\beta_i U_{0i} + (1 - \beta_i)\overline{U})\Big]. \tag{2}$$

The relevant incentive constraints for the producer are as follows:

$$U_{1i} \geq w_{1i} + \delta\overline{U}, \tag{P-IC-e}$$

$$U_{1i} \geq \overline{U}, \tag{P-IC1}$$

pairwise stability restriction is relaxed.

$$U_{0i} \geq \overline{U}. \tag{P-IC0}$$

Constraint (P-IC-e) requires the producer to choose effort over shirking when the service is needed. Constraints (P-IC1) and (P-IC0) require that the producer remain with the current buyer when the demand is 1 or 0, respectively. If $\beta_i = 0$, equation (2) and constraint (P-IC0) do not apply, as the buyer immediately separates from the producer when demand is zero.

For the buyer, the post-matching continuation payoffs when $d_{it} = 1$ and $d_{it} = 0$ are

$$\Pi_{1i} = y - w_{1i} + \delta\Big[(1-\alpha_{1i})\Pi_{1i} + \alpha_{1i}(\beta_i\Pi_{0i} + (1-\beta_i)\overline{\Pi}_{0i})\Big],$$

and

$$\Pi_{0i} = -w_{0i} + \delta\Big[\alpha_{0i}\Pi_{1i} + (1-\alpha_{0i})(\beta_i\Pi_{0i} + (1-\beta_i)\overline{\Pi}_{0i})\Big].$$

where $\overline{\Pi}_{1i}$ and $\overline{\Pi}_{0i}$ is the value of the buyer's pre-matching continuation values when $d_{it} = 1$ or $d_{it} = 0$, respectively. Since there is an excess of producers in the frictionless matching market, the buyer can always successfully find a match, so $\overline{\Pi}_{1i} = \Pi_{1i}$.

The relevant incentive constraint for the buyer is

$$\Pi_{1i} \geq \delta(\alpha_{1i}\overline{\Pi}_{0i} + (1-\alpha_{1i})\overline{\Pi}_{1i}), \tag{B-IC-w}$$

$$\Pi_{1i} \geq \overline{\Pi}_{1i}, \tag{B-IC1}$$

$$\Pi_{0i} \geq \overline{\Pi}_{0i}, \tag{B-IC0}$$

Constraint (B-IC-w) ensures that the buyer honors the payment to the producer. Constraints (B-IC1) and (B-IC0) reflect the buyer's desire to retain the producer when there is a demand or not, respectively. As before, if $\beta_i = 0$, the term Π_{0i} and constraint (B-IC0) do not apply, as the buyer would immediately separate from the producer.

The optimal bilateral contract is obtained by choosing w_{1i}, w_{0i}, and β_i to maximize Π_{1i}, subject to (B-IC-w), (B-IC1),(P-IC-e), (P-IC1), as well as (B-IC0) and (P-IC0) if $\beta_i > 0$.

Lemma 1. *Suppose an optimal bilateral relational contract exists. Under this contract, if*

$$\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_{0i}}{1-\alpha_{1i}}, \tag{3}$$

then the buyer pays

$$w_{BS}(\alpha_i) = \left(\frac{1}{\delta}\frac{1}{1-\alpha_{1i}}\right)c + (1-\delta)\overline{U}. \tag{4}$$

when demand is one and separates from the producer when demand becomes zero. Otherwise, the buyer pays

$$w_{BI}(\alpha_i) = \left(\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_{0i}}{(1-\alpha_{1i})+\frac{\delta}{1-\delta}\alpha_{0i}}\right)c + \left(\frac{1+\frac{\delta}{1-\delta}\alpha_{0i}}{(1-\alpha_{1i})+\frac{\delta}{1-\delta}\alpha_{0i}} - \delta\right)\overline{U} \tag{5}$$

when demand is one, remains matched with the idle producer when demands switches to zero, and pays zero until demand switches back to one.

Proof. All omitted proofs are in the appendix. □

Lemma 1 shows that the optimal bilateral contract features a buyer who either always separates from a producer or always retains him when their demand switches to zero. According to Equation (3), bilateral contracts with separation dominates bilateral contracts with idleness when (a) the producer's continuation value when unmatched $\overline{U}$ is high, (b) the buyer has a smaller α_{0i}, so a longer period of idleness is expected, and (c) the buyer has a large α_{1i}, so a shorter period without idleness is expected.

The payment to producers in a bilateral contract, $w_B(\alpha_i) \equiv \min\{w_{BS}(\alpha_i), w_{BI}(\alpha_i)\}$, is increasing in α_{1i}. This is because when business needs are shorter-lived, the producer faces a higher chance of either separating or becoming idle, so the future surplus in the relationship is smaller. A higher incentive rent is therefore needed to incentivize producer effort.

3.2 Intermediated Relational Contracts

Under intermediated contracts, buyers delegate to intermediaries the responsibility of motivating and monitoring producers. The intermediary fulfills the buyers's demand by entering relational contracts with a large number of producers and assigning producers to buyers according to demand realization.

We first analyze the contract that intermediaries offer to producers. To meet buyer demand, we assume that each intermediary offers intermediary-optimal, contract-specific, and stationary contracts to producers. As shown in Section 3.1, the terms in an optimal bilateral contract with producer hinge on the buyer's demand-switching probabilities. Unlike buyers, however, intermediaries face constant demand for services and therefore have constant demand for effort from its matched producers. The reason is that each intermediary randomly matches with a continuum of buyers drawn from the same distribution, so by the law of large numbers, the intermediaries face total demand from buyers that is constant over time. Anticipating this stable demand for services, the measure of producers that each intermediary contracts with is equal to the expected measure of demand realizations, and each matched producer is asked to exert effort in every period. Therefore, by the logic of Lemma 1, the compensating payment for the producer is $w_{1i} = w_M$ in every period, where

$$w_M = \frac{c}{\delta} + (1 - \delta)\overline{U}. \tag{6}$$

We next consider how producers are assigned to buyers in each period under the intermediated contract. Note that buyers are indifferent between any assignment of producers where the assigned producer exerts effort, since producers are identical in our model. Producers are also indifferent between any assignment of buyers where the intermediaries offer the same level of payments. Since intermediaries require producers to exert effort and provide the same compensating payment in every period, buyers and producers have the same payoffs in any assignment where producers are matched with buyers with positive demand in every period. Therefore, any such assignment is optimal.

We can now characterize the optimal intermediated contract. Let $C_i^M = (p_{1i}, p_{0i}, \beta_i)$ denote a buyer-optimal, contract-specific, and stationary intermediated relational contract offered by a buyer to an intermediary. Here p_{1i} and p_{0i} are the service fees when the buyer needs and does not need the service, respectively. If the buyer deviates from the specified service fee, the intermediary does not assign a producer to the buyer and separates from the buyer. If instead the producer assigned by the intermediary deviates from the specified effort, the buyer separates from the intermediary after production. Since matching is not anonymous in the intermediary market, the buyer never chooses to match with the defaulted intermediary again in the future.[16]

Under C_i^M, the post-matching continuation payoffs for the intermediary in a buyer-intermediary match, when the service is and is not needed, respectively, are

$$V_{1i} = p_{1i} - w_M + \delta\Big[(1-\alpha_{1i})V_{1i} + \alpha_{1i}(\beta_i V_{0i} + (1-\beta_i)\overline{V})\Big], \tag{7}$$

$$V_{0i} = p_{0i} + \delta\Big[\alpha_{0i}V_{1i} + (1-\alpha_{0i})(\beta_i V_{0i} + (1-\beta_i)\overline{V})\Big], \tag{8}$$

where $\overline{V}$ is the value of an intermediary's continuation value after separating from a buyer. The relevant incentive constraints for the intermediary, similar with those for a producer, are

$$V_{1i} \geq p_{1i} + \delta\overline{V}, \tag{M-IC-w}$$

$$V_{1i} \geq \overline{V}, \tag{M-IC1}$$

$$V_{0i} \geq \overline{V}, \tag{M-IC0}$$

Note that $\overline{V} = 0$ on the equilibrium path. If an intermediary separates from a buyer, it cannot match with a new buyer, because all other potential buyers are matched with some intermediary and will not become unmatched on the equilibrium path.

For the buyer, the continuation payoffs and incentive constraints are the same as in the bilateral contract, except that the payments w_{0i} and w_{1i} to the producer are replaced with service fee payments p_{0i} and p_{1i} to the intermediary. For concision, we omit these conditions, which simply repeat (B-IC-w), (B-IC-1), and (B-IC0). The optimal intermediated contract maximizes Π_{1i} subject to these incentive compatibility constraints.

[16] We assume that in the intermediary market, buyers randomly match with an intermediary who has not breached the relational contracts with them. Specifically, let $\mathbf{K}_{i,t}$ be the set of intermediaries who have interacted with buyer i and breached the relational contract with i before period t. Should a buyer become unmatched and match with an intermediary in the intermediary market in period t, she randomly matches with one of the intermediaries from the set $\mathbf{K} \setminus \mathbf{K}_{i,t}$.

Lemma 2. *Suppose an optimal intermediated contract exists. Under this contract, the buyer always retains the intermediary, pays zero service fees to the intermediary when there is no demand, and when there is demand, she pays the intermediary a service fee equal to*

$$p(\alpha_i) = \left(\frac{1}{\delta} \frac{1 + \frac{\delta}{1-\delta}\alpha_{0i}}{(1 - \alpha_{1i}) + \frac{\delta}{1-\delta}\alpha_{0i}} \right) w_M. \tag{9}$$

Lemma 2 shows that the cost of intermediated contract is a form of double marginalization. Note that $p(\alpha_i)$ can be rewritten as the product of $\lambda(\alpha_i) = \frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_{0i}}{(1-\alpha_{1i})+\frac{\delta}{1-\delta}\alpha_{0i}}$ and the payment to the producer w_M. Here $\lambda(\alpha_i)$ can be thought of as an intermediary markup. Furthermore, as shown in Equation (6), w_M is elevated above the cost of effort to the producer. In other words, both the intermediary and producer are paid rents so that both are incentivized to honor their contractual obligations.

The benefit of intermediated contract is that the payment to the producer is lower than the payment under bilateral contracts, since the intermediary can smooth demand across its buyers. To see this, note that $w_B(\alpha_i) \geq w_M$ for all α_i, where equality holds if and only if $\alpha_{1i} = 0$.

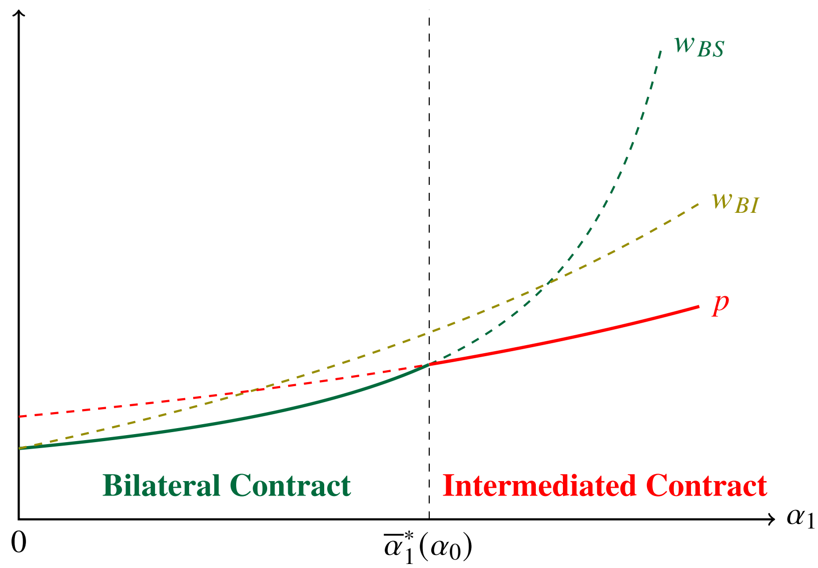
3.3 Optimal Contractual Choice

Having characterized bilateral and intermediated contracts, we now characterize when buyers choose intermediation. We focus on buyers who have the same α_{0i}, which is greater when a buyer's demand quickly returns to one from zero. We ask how the optimal contractual choice depends on α_{1i}, which is greater when the spells where a buyer have demand are short-term, and $\overline{U}$, which is greater when producer entry is costly, so the producer market is tight and it is easy for producers to rematch.

To compare the two arrangements, it suffices to compare the payment $w_B(\alpha_i)$ under bilateral contracting and the service fee $p(\alpha_i)$ under intermediation. This is because the buyer pays nothing when their demand is zero and can always match with a producer when her demand is one.

Figure 2 provides a graphical illustration of this comparison. For a buyer with stable demand,

Figure 2: Service cost under bilateral and intermediated contracts



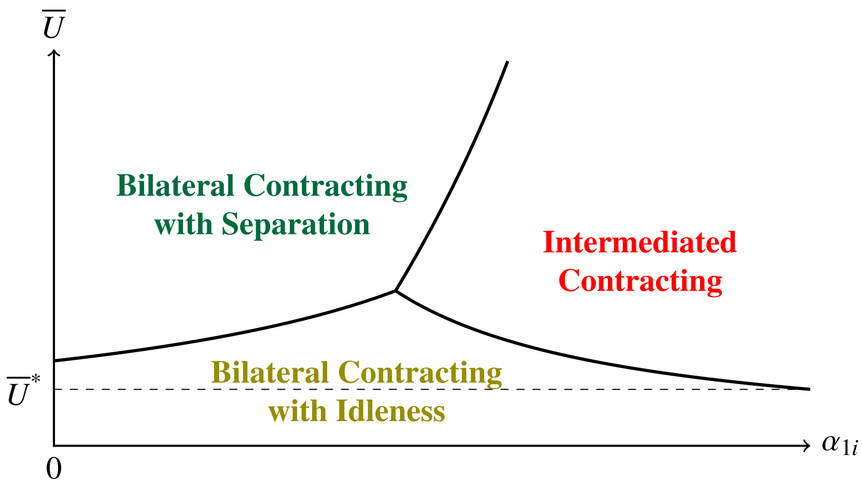
Note: α_1 is the probability that the buyer's demand switches from 1 to 0.

i.e., for whom $\alpha_{1i} = 0$, intermediated contracting is strictly more expensive than bilateral contracting because of double marginalization. To obtain high-quality services, the buyer needs to pay additional rent to the intermediary. However, there is no benefit to intermediation, since demand is stable, so the producer is paid the same incentive rent under bilateral contracting.

The advantage of intermediation over bilateral contracting becomes larger when business needs are short-term. As α_{1i} becomes larger, producers must be paid elevated payments in order for them to exert effort. Since intermediaries can reassign producers across buyers based on demands so producers neither separate nor become idle, the cost of incentivizing producers is lowered. To see this mathematically, note that $w_{BS}(\alpha_i)$ approaches infinity as α_{1i} approaches one. Furthermore, $p(\alpha_i)$ increases in α_{1i} less steeply than $w_{BI}(\alpha_i)$ if c is relatively small and $\overline{U}$ is relatively large. Therefore, when rematching is easy, the intermediary operates a more cost-efficient internal producer market by having long-standing relational contracts on both sides of the market.

Figure 3 graphically shows the optimal contracts as a function of α_{1i} and $\overline{U}$. In this figure, provided $\overline{U} > \overline{U}^*$, then there exists a cutoff value such that intermediated contracts dominates if and only if α_{1i} is sufficiently large. If $\overline{U} < \overline{U}^*$, then the producer's pre-matching continuation

Figure 3: Optimal contractual choice given model parameters



value is low, so it is optimal to retain them and keep them idle until demand returns.

The following proposition formalizes this result.

Proposition 1. *Take any set of buyers i with the same α_{0i} for whom an optimal contract exists. There exists $\overline{U}^*$ such that:*

1. *If $\overline{U} < \overline{U}^*$, a bilateral contract is optimal for all i in this set;*
2. *If $\overline{U} > \overline{U}^*$, there exists $\alpha_1^*(\alpha_{0i}) \in (0, 1)$ such that an intermediated contract is optimal if and only if $\alpha_{1i} \geq \alpha_1^*(\alpha_{0i})$.*

Proposition 1 can be viewed a new answer to the question first posed by Coase (1937): Why and under what conditions should we expect centralized allocators to emerge in decentralized markets? The above result shows that intermediation dominates bilateral pairings when business needs are short-term and the continuation value of an unmatched producer is high. This is true even though buyers and producers in the model can frictionlessly meet and enter contracts. Our result therefore implies that centralized allocators may emerge in the absence of many types of transaction costs, including search, bargaining, and contract-writing costs.

That buyers with frequent demand are more likely to disintermediate has been frequently remarked upon in related literature. Williamson (1985) attributes this tendency to administrative and bargaining costs in repeated transactions. Wernerfelt (2015, 2016) attributes it to switching costs.[17] In our model, the cost of intermediation is instead a price premium charged by the intermediary to ensure its performance as an aggregator, monitor, and contract enforcer.

4 Steady-State Equilibrium

In this section, we show that there exists a unique steady-state equilibrium. We then explore the model's empirical implications. First, we compare the outcomes of intermediated and bilaterally contracted producers in equilibrium. Second, we compare the outcomes and welfare of buyers and producers in economies with and without intermediaries. Third, we study how intermediation choices depends on market size.

4.1 Deriving the Equilibrium

We say that the economy is in a steady state when (1) the number of producers in the economy and the distribution of contracts in the matching market are unchanging across periods and (2) each buyer's demand evolves according to its long-run distribution. By the properties of Markov processes, the steady-state probability that a buyer i has positive demand in any period is equal to $\pi_i = \alpha_{0i}/(\alpha_{0i} + \alpha_{1i})$.

At the start of each period, some buyers and intermediaries are unmatched and directly offer bilateral contracts to unmatched producers. Let $\mathcal{I}_{BS}$ denote the set of buyers who enter bilateral contracts that end when their demand switches to zero. For a buyer $i \in \mathcal{I}_{BS}$, the steady-state probability that they are unmatched is

$$v_i = (1 - \pi_i)\, \alpha_{0i}. \tag{10}$$

Let $\mathcal{I}_{BI}$ be the set of buyers who directly contract with producers in contracts that continue when demand switches. Let $\mathcal{I}_M$ be the set of buyers who contract with intermediaries. For both types of contracts, producers never separate. Therefore, for any buyer $i \in \mathcal{I}_{BI} \cup \mathcal{I}_M$, the steady-state probability that they are unmatched is $v_i = 0$. The total measure of new contracts

[17] See also Demsetz (1988).

offered in the producer market is given by

$$v = \int_{\mathcal{I}_{BS}} v_i dF. \tag{11}$$

The steady-state measure of producers in bilateral contracts, n_B, is given by

$$n_B = \int_{\mathcal{I}_{BS}} \pi_i dF + \int_{\mathcal{I}_{BI}} dF. \tag{12}$$

The steady-state measure of producers in intermediated contracts, n_M, is given by

$$n_M = \int_{\mathcal{I}_M} \pi_i dF. \tag{13}$$

The measure of unmatched producers after matching is

$$n_N = n - n_B - n_M. \tag{14}$$

The value of entering either bilateral or intermediated contracts is higher than the value of being unmatched, so there is an excess of producers who enter the producer market. This implies that $n_N > 0$.

Since matching is frictionless, all contracts offered in the producer market are immediately filled. The total measure of unmatched producers before matching, u, is given by

$$u = v + n_N. \tag{15}$$

Matching is random and contract offers are never made to matched producers, so the Bellman equation for an unmatched producer is given by

$$\overline{U} = \int_{\mathcal{I}_{BS}} \frac{v_i}{u} U_{BS}(\alpha_i) dF + \left(1 - \frac{v}{u}\right) \delta \overline{U}, \tag{16}$$

where $U_{BS}(\alpha_i) = w_{BS}(\alpha_i) + \delta\overline{U}$. Note that the continuation value of being unmatched ($\overline{U}$) consists of two components. The first term reflect the value of matching with an unmatched buyer. The second term reflects the value of remaining unmatched.

To close the model, we solve for the steady-state number of producers that enter the economy. By assumption, producers enter the economy at the beginning of each period by paying an entry cost C. Entry drives down the likelihood that producers are matched, so they enter only until the continuation value of being unmatched in the labor market equals their entry cost. This yields the following condition:

$$\overline{U} = C. \tag{17}$$

We can now define an equilibrium in our economy.

Definition 1. *A steady-state equilibrium is a distribution of contracts offered by each buyer and intermediary such that:*

1. *All contracts are offerer-optimal, contract-specific, and stationary;*
2. *Each player's pre-matching continuation value is determined by steady-state transition probabilities and frictionless and random matching via Equation* (16)*;*
3. *The measure of producers in the economy is derived from the producer entry condition, given by Equation* (17)*.*

Proposition 2. *There exists a unique steady-state equilibrium.*

Proof. By Equation (17), $\overline{U}$ equals the entry cost C. Given $\overline{U}$, we can compare the values of $w_{BS}(\alpha_i)$, $w_{BI}(\alpha_i)$, $p(\alpha_i)$, and y for each i using Equations (4), (5), and (9) to determine $\mathcal{I}_{BS}$, $\mathcal{I}_{BI}$, and $\mathcal{I}_M$. Having derived these, we can obtain unique values for v_i, v, n_B, and n_M from Equations (10), (11), (12), and (13). We plug these into Equation (16) to solve for a unique value for u. Plugging u into Equations (14) and (15) then yields a unique value for n. □

4.2 Empirical Implications

Having shown that there exists a unique steady-state equilibrium in this exchange economy, we can now explore the model's predictions regarding equilibrium behavior.

Effects of Intermediation on Producers

We first compare the pay, separation rates, and idleness of bilterally contracted and intermediated producers in the steady-state equilibrium. We focus on the interesting case where all contract types coexist,[18] and derive four findings.

First, bilaterally contracted producers in our model have *higher average pay* per unit effort than intermediated producers. This follows from the fact that $w_B(\alpha_i) > w_M$ for all buyers i with fluctuating demand.

Second, bilaterally contracted producers have *more dispersed pay* per unit effort than those of intermediated producers. This is because $w_B(\alpha_i)$ takes on different values depending on α_i, which is heterogeneous across buyers, while w_M is constant.

Third, bilaterally contracted producers have *higher separation rates*. We define a producer's separation rate as the probability that a matched producer becomes unmatched at the start of the next period. This probability is higher for bilaterally contracted producers, since they may separate from buyers when their demand changes, while intermediated producers are always reallocated among the intermediary's clients.

Fourth, bilaterally contracted producers are *more likely to be idle* during their employment spells. We define idleness as the steady-state probability that a producer is matched but does not exert effort. In our model, intermediated producers are never idle, while bilaterally contracted producers may have positive idleness.

Corollary 1. *Compared to intermediated producers, bilaterally contracted producers have higher average pay, more dispersed pay, higher separation rates, and higher idleness in equilibrium.*

Trade and Welfare with and without Intermediaries

We next analyze how the presence of intermediaries alters the welfare of buyers and producers. We find that intermediaries benefit some buyers, but on average reduce producer rents.

To show this, we consider two economies: one with intermediaries and one without. We assume that producers freely enter at some exogenous cost C in both economies. Therefore, the

[18] Specifically, we assume that y, C and F are such that $|\mathcal{I}_M|, |\mathcal{I}_{BI}|, |\mathcal{I}_{BS}| > 0$. If F has full support on $[0,1] \times [0,1]$, then by Proposition 1 there exists y and C such that this holds.

producer's continuation values when unmatched, and hence the contractual terms offered to them by buyers under bilateral contracting, are the same in the two economies.The only difference is the added possibility of intermediated contracts.

The introduction of intermediaries may cause three types of buyers to switch to intermediation: buyers who do not initially consume services, buyers who initially choose bilateral contracts with idleness, and buyers who initially choose bilateral contracts with separation. We denote the three subsets of switching buyers as $\mathcal{S}_0$, $\mathcal{S}_I$, and $\mathcal{S}_S$.[19] The contracting choices of the remaining buyers are unchanged. We focus on the case where the measure of buyers switch from no consumption to intermediation and the measure of buyers switch from bilateral contracts to intermediation are both nonzero.[20]

It follows easily that the introduction of intermediaries benefit buyers with fluctuating demand, but it hurts producers who initially earning pay premiums. By Corollary 1, the average producer pay per unit effort falls. Producer separation rates and idleness also fall. At the same time, the per-period payoff of buyers who switch to intermediation increases, while the per-period payoff of the remaining buyers are unchanged.

The introduction of intermediaries also unambiguously increases the measure of buyers who receive services. However, it is ambiguous whether the measure of matched producers ($n_M + n_B$) increases or falls. On one hand, intermediaries enable more buyers to afford services, which increases demand for producers. On the other, intermediaries reduce idleness, so fewer producers are needed to fulfill the same level of demand. The relative magntitude of these two effects depends on the distribution of buyer types.

Corollary 2. *When intermediaries are introduced into an economy, the payoffs of some buyers increase, while the payoffs of the remaining buyers are unchanged. The mean pay per unit effort received by producers falls. The measure of buyers who receive services increases. The total measure of matched producers increases if and only if* $\int_{\mathcal{S}_0} \pi_i dF > \int_{\mathcal{S}_I} (1 - \pi_i) dF$.

[19] Assuming that indifferent buyers do not switch, we have that $\mathcal{S}_0 = \{i \mid p(\alpha_i) < y < w_B(\alpha_i)\}$, $\mathcal{S}_I = \{i \mid p(\alpha_i) < w_{BI}(\alpha_i) < \min\{y, w_{BS}(\alpha_i)\}\}$, and $\mathcal{S}_S = \{i \mid p(\alpha_i) < w_{BS}(\alpha_i) < \min\{y, w_{BI}(\alpha_i)\}\}$.

[20] Formally, we assume that y, C and F are such that $|\mathcal{S}_0| > 0$ and $|\mathcal{S}_I \cup \mathcal{S}_S| > 0$. This condition requires that there exists $\alpha_i, \alpha_i' \in \operatorname{supp} F$ such that $y > w_B(\alpha_i) > p(\alpha_i)$ and $w_B(\alpha_i') > y > p(\alpha_i')$. If F has full support on $[0, 1] \times [0, 1]$, then by Proposition 1 there exists y and C such that this holds.

Intermediation is Limited by the Extent of the Market

An important feature of our model is that intermediation is efficiency-enhancing only if there are multiple sellers and buyers. Suppose, as an extreme case, that there is only one buyer and one producer in the economy. In such a case, it is never profitable for a buyer to use an intermediary, since the intermediary will require a markup and the cost savings from demand aggregation only emerge when the market is sufficiently big. Therefore, intermediation is more likely when *the extent of the market* — i.e. the available number of buyers and sellers — is large. This result is closely related to Stigler (1951), who conjectures that firms spin-off production stages because of increasing economies of scale as the market grows.

Remark 1. *In an economy with a single buyer and a single producer, intermediaries are always matched with zero buyers and zero producers.*

5 Extensions

This section extends the model to study the relationship between intermediation, producer specialization, and intermediary reputation concerns. We first incorporate an endogenous choice by producers to specialize in different capabilities upon entry into the economy. Then, we add the possibility that poor performance by an intermediary is made known to a broader set of buyers.

5.1 Specialization

We consider a two-task economy with a measure of buyers i who have unit demand in every period, for either one of two tasks $d_{it} \in \{A, B\}$. Each buyer's demand switches from one task to another task with some symmetric probability α_i at the start of each period.[21] There is also an excess measure of producers who choose whether to become either specialists in one of two activities needed by buyers or a generalist with middling skill in both services. Let $\phi_j \in \{A, B, G\}$ denote the chosen type of the producer, where A and B refer to specialists and G refers to the generalist.

[21] This is essentially simplifying the model in Section 2 by assuming $\alpha_{Ai} = \alpha_{Bi} = \alpha_i$.

The output depends on the buyer's demand d_{it}, the producer's type ϕ_j, and the producer's chosen effort e_t, and is given by

$$y_{it} = \left[y \cdot \mathbf{1}\{\phi_j = G\} + (y + \Delta_i) \cdot \mathbf{1}\{\phi_j = d_{it}\} \right] e_t,$$

where $\Delta_i > 0$ denotes the buyer-specific *gains from specialization*. If the specialist exerts effort, output is high when demand and the producer's type are well-matched, but low when they are not. Output is always middling for generalists who exert effort.

As before, neither pay w_t nor effort e_t are contractible and must be incentivized through relational contracts. We assume that output y is sufficiently large so that buyers are always able to receive positive profit by bilaterally contracting with a generalist, so there are never buyers who do not receive services. We also assume that each producer's entry cost C is sufficiently large so that specialist producers never remain in a contract but become idle when the demand of their buyer changes. These assumptions allow us to focus on each buyer's choice between directly contracting with a generalist, directly contracting a specialist who is never idle, and intermediation. There are K intermediaries for each task. Each intermediary can only contract with producers specializing in that task and are randomly matched with entreprenuers who offer intermediated contracts.[22]

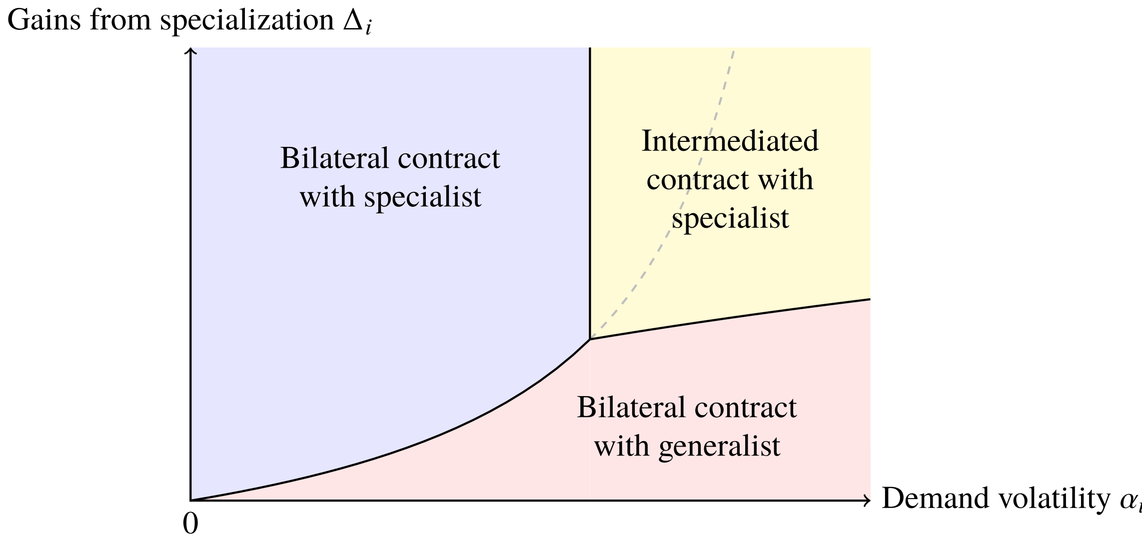
For buyers with large α_i and large Δ_i, intermediation dominates either bilteral contracting arrangement. Due to double marginalization, however, it is optimal to directly contract with producers if demand is volatile and gains from specialization are small. It is also never optimal to enter an intermediated contract wherein the intermediary contracts with a generalist producer and sends the producer to different clients. Therefore, some buyer choose intermediated contracts with specialists, while others bilaterally contract with either specialists or generalists.

Corollary 3. *In the unique steady-state equilibrium of a two-task economy, buyer i chooses intermediated contracts if and only if their demand volatility α_i and gains from specialization Δ_i are both sufficiently large.*

Another implication is that the presence of intermediaries encourages producers to specialize.

[22]This setup implicitly assumes that each intermediary specializes in monitoring one type of task.

Figure 4: Optimal contractual choice in the presence of gains from specialization



In the absence of intermediaries, buyers with greater α_i and hence more volatile demand contract with generalists, while those with smaller α_i and hence less volatile demand contract with specialists. The dashed curve in Figure 4 shows the boundary between contracting with a specialist and with a generalist in the absence of intermediation. When intermediation becomes possible, however, their demand for directly contracted generalists is replaced with demand for intermediated specialists. This causes the overall demand for specialists to rise. In response, more producers choose to become specialists. Correspondingly, there are fewer bilateral contracts with elevated pay, so fewer producers enter, and the measure of unmatched producers falls. To show this formally, we assume that (α_i, Δ_i) are drawn from a distribution G that a non-zero measure of buyers choose intermediated contracts, and that the conditional distribution of α_i given Δ_i has positive support on $[0, 1]$.

Corollary 4. *When intermediaries are introduced into a two-task economy, the measure of specialist producers increase and the measure of unmatched producers falls.*

5.2 Reputation Concerns

Thus far, our model follows Shapiro and Stiglitz (1984) in assuming that producers are motivated to perform through the threat of contract termination, which would require them to wait to rematch in an anonymous matching market. Another source of motivation, as modeled by Klein and Leffler (1981), is that producers may lose reputational capital when they renege on promises to deliver high-quality services, causing a broader set of buyers to withhold future business.

In this subsection, we show that reputation concerns can play an important role in shaping the choice between intermediation and bilateral contracting. In particular, we show that intermediation becomes cheaper if the performance of the intermediary is partially observable to outside parties.

To see this formally, we consider a two-task economy with reputable intermediaries, where low effort by a producer is communicated with some probability to other buyers who can withhold future business from the producer's matched intermediary. Specifically, we assume that with probability $\gamma \in [0, 1]$, the effort choice by a producer contracted by the intermediary is observed by another buyer, who is drawn among all buyers with uniform probability. The parameter γ measures the *ease of word-of-mouth communication* as enabled by communication technologies such as the Internet and social media platforms.[23]

As γ increases, the intermediary faces a harsher punishment if it reneges on its contracts. To see this, note that an intermediary's mean continuation value from being matched with an buyer is $\tilde{V} = \frac{1}{|\mathcal{I}|} \int_{\mathcal{I}_O} \pi_i V_{1i} + (1 - \pi_i) V_{0i} dF > 0$. When $\gamma > 0$, the intermediary's binding IC constraint is now given by:

$$V_1 \geq p_1 + \delta(-\gamma \tilde{V}), \tag{M-IC-w'}$$

In other words, the threat of multilateral punishment means that a reduced mark-up is needed to incentivize the intermediary to perform. The unit cost of intermediated service therefore falls as γ increases.

As the cost of intermediation decreases, a larger measure of buyers choose intermediation. Correspondingly, the measure of buyers who directly contract with specialists and generalists both fall. This enables greater producer specialization, since total demand for specialized producers increases. At the same time, there are fewer bilateral contracts with elevated pay, so the measure of unmatched producers falls.

[23]Here the intermediary will not wish to renege on more than one of its clients if she does not wish to do so for a single client, since an buyer may learn of bad service provided to multiple clients from word-of-mouth communication but can only punish maximally once.

Corollary 5. *Consider a two-task economy with reputable intermediaries. As the ease of word-of-mouth communication increases, the measure of intermediated producers increases, the measure of specialist producers increases, and the measure of unmatched producers falls.*

Remark. By construction, our model disallows two possibilities that may in reality affect intermediation choices. First, producers themselves may build and maintain reputational capital. If so, they would not rematch on an *anonymous* market, as modeled above. Instead, if a producer shirks their contractual responsibilities, buyers can cause producers to face difficulty rematching thereafter. In this case, the producer faces a stronger incentive to perform and does so even if the particular buyer they currently provide services for is expected to have no demand next period. Therefore, intermediation becomes less attractive if producers maintain reputational capital.

Second, producers matched with the intermediary may be able to communicate with one another about the intermediary's actions, such as through regular business conferences. This opens up the possibility that producers collectively punish intermediaries in response to contract infringement (a la Levin 2002). In our model, the threat of losing multiple producer relationships has no bite because intermediaries can immediately rematch with new producers. However, if rematching with producers is assumed to be difficult for intermediaries, then multilateral relational contracts between an intermediary and its producers will increase the intermediary's incentive to perform, thereby lowering its service fee. Intermediation therefore becomes more attractive if producers can collectively punish intermediaries.

6 Applications

There are many examples of real-world intermediaries that engage in behavior consistent with our model. Table 1 provides a list. In this section, we discuss professional service outsourcing in detail and compare model predictions with empirical findings in the literature. We then briefly discuss a few further examples, including franchises, online platform, and hospitals.

Professional service outsourcing. Consider an entrepreneur who needs a service performed. She can fulfill the demand either by employing an in-house worker or contracting it out to an external intermediary firm. Entrepreneurs may make these employ-or-outsource decisions for many types of professional services, including cleaning, security, accounting, legal, IT, and HR services.[24] These decisions determine the labor boundary of firms.

Our model provides a novel lens with which to understand these decisions. We can define *employment* as a bilateral relational contract between an employer and a producer and *outsourcing* as an intermediated contract in which an intermediary employs workers and assigns them to different entrepreneurs. Under these definitions, the model generates predictions for the determinants and effects of professional service outsourcing that are consistent with evidence.[25]

Proposition 1 and Corollary 3 suggest that outsourcing is more likely when demand is volatile and gains from specialization are large. Consistent with this, Abraham and Taylor (1996) find that establishments with cyclical demand and specialized needs are more likely to outsource accounting services. Houseman (2001) similarly finds that the need to accommodate fluctuations in workload is a commonly cited reason for using flexible staffing arrangements.

Corollary 1 shows that outsourced workers earn lower and less dispersed wages, and are less likely to become unmatched. Consistent with this, several studies show that outsourced workers earn lower and more compressed wages (Dube and Kaplan 2010; Goldschmidt and Schmieder 2017; Drenik et al. 2020). Outsourced workers are also documented to have lower hazard into unemployment than comparable direct employees (Guo, Li and Wong 2024).

Remark 1 and Corollary 4 show that market size increases intermediation, which in turn enables specialization. Consistent with this, Garicano and Hubbard (2009) find that both the share of lawyers that specialize and the share of lawyers working in specialized firms increases

[24]Professional service firms account for 12 percent of U.S. employment and are among the largest employers in the world (Berlingieri 2013). More broadly defined, services account for the vast majority of cross-establishment trade (Bostanci and Kambhampati 2022).

[25]Our view of the intermediary as a nexus of relational contracts is closely related to an early strand of the theory of the firm, which argues that the firm is no more than a nexus of contracts (Alchian and Demsetz 1972; Jensen and Meckling 1976; Cheung 1983; Demsetz 1988). In contrast to our view, recent literature on the theory of the firm tends to emphasize the role of property rights (e.g., Grossman and Hart 1986; Gibbons 2005) and transaction costs arising from asset specificity, bounded rationality, bargaining frictions, or contract-writing costs (e.g., Bajari and Tadelis 2001; Tadelis 2002; Hart and Moore 2007, 2008; Levin and Tadelis 2010; Tadelis and Williamson 2012; Wernerfelt 2015, 2016). These views, however, do not easily explain the empirical facts discussed in this section.

with market size.[26]

Corollary 5 shows that ease of word-of-mouth communication increases intermediation and specialization and reduces unemployment. Consistent with this, Bergeaud et al. (2021) provide causal evidence that the rise of broadband internet increased outsourcing and increased the homogeneity of occupations within firms. Furthermore, firms are increasingly outsource their non-core activities, workers and firms have become increasingly specialized, and unemployment has fallen over the past half century (Katz and Krueger 1999; Weil 2014; Handwerker 2023). This may be partly explained by the arrival of communication technologies like the Internet and social media, which has increased the role of branding and reputation in facilitating trade.

Franchises. Retail chains are often operated as franchises, in which a franchisor creates a branded good or service that is distributed by locally owned and operated franchisees.[27] The franchisor can be viewed as a centralized intermediary that maintains relational contracts with both franchisees and customers, and directs customers to franchised stores, and supervises franchisees to provide quality services to customers. By Proposition 1, airports filled with itinerant travelers should feature more franchised stores, while small towns with immobile populations are more likely to have independent operations.

Online platforms. Online platforms such Uber operate feedback and recommendation systems that direct exchange and incentivize performance. Their function is similar to the allocative and monitoring tasks performed by human managers in professional service firms.[28] Online platforms can therefore be viewed as centralized relational intermediaries that aggregate demand

[26] For related evidence, see also Baumgardner (1988*a*,*b*) and Duranton and Jayet (2011).

[27] Examples include fast food restaurant chains like McDonald's, Starbucks, and Dunkin Donuts, hotel chains like Marriott, home health care chains such as BrightStar, and business service chains like UPS. A related form of franchising involves an upstream manufacturer and a downstream retailer who sells the good, such as gasoline or automobiles. Klein (1995) provides a related repeated-game model of franchise contracts. See also Blair and Lafontaine (2005), who provide a comprehensive empirical analysis of franchises and franchise contracts.

[28] The economic analysis of online platforms largely focuses on platform pricing in the presence of usage and membership externalities (Rochet and Tirole 2003, 2006) and the impact of platforms on consumer search (Brynjolfsson and Smith 2000; Ellison and Ellison 2009). Less attention has been devoted to understanding how these platforms incentivize platform participants to perform. A notable exception is Hagiu and Wright (2015), who model the organizational difference between vertical integration and multi-sided platform as a difference in the allocation of decision rights, following Gibbons (2005).

for a large number of customers, direct producers, and ensure performance. By Proposition 1, an occasional rider should call for an Uber ride, but a company with persistent demand will prefer to directly employ a full-time driver instead.

Hospitals. Patients fall sick randomly and intermittently, possibly with different medical needs each time. It is efficient to assign patients to different doctors and nurses, who specialize in specific medical procedures and knowledge. This can give rise to hospital organizations that aggregate the demand for medical services, assign the appropriate medical practitioners to provide services, and ensure that patients are provided with quality care. By Proposition 1, independent practices run by a single doctor will tend to be more prevalent in small towns. By Corollaries 1 and 3, independent practices tend to be generalist and are more susceptible to idleness.

7 Conclusion

According to a large and multi-disciplinary literature, intermediaries that maintain relational contracts and coordinate transactions are ubiquitous and essential for trade. In this paper, we develop a repeated-game model of an exchange economy to explore the drivers and consequences of relational intermediation. We show that intermediaries can redress moral hazard by entering relational contracts with a multitude of buyers and producers and directing trade between them.

Our main propositions show that buyers sort into bilateral and intermediated contracts in the unique steady-state equilibrium: buyers with sufficiently volatile demand choose intermediation, while buyers with long-lasting demand choose bilateral contracts. The reason is that intermediaries are useful only when demand is short-lived, and a markup is needed to incentivize the intermediary to perform.

Many empirical implications about the determinants and effects of intermediation are derived. We show that the optimal choice between intermediation and bilateral contracting depends on demand volatility, gains from specialization, market tightness, the extent of the market, and reputational effects. We characterize the effects of intermediation on output, welfare, the patterns of specialization, and the distribution of rents in the economy. In contrast to models of intermediaries based on search frictions and asymmetric information, we show that falling communication costs can increase intermediation.

In our main application, we use the model to explain empirical findings regarding the determinants and effects of professional service outsourcing. As such, our work sheds new light on classic questions regarding the boundaries of firms that were first raised by Coase (1937). Our model can also used to understand the structure of supply chains, franchises, online platforms, and many other intermediary organizations. Given its tractability and applicability, we believe that our model is a useful addition to the economist's tool set for analyzing market microstructure and the role of intermediaries in trade.

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A Appendix

We omit the subscript i in the proofs. We first establish the following two lemmas.

Lemma A.1. *Suppose an optimal bilateral relational contract exists. Under this contract, if $\beta > 0$, then $w_0 = 0$, both (B-IC1) and (B-IC0) bind, and (P-IC1) is slack.*

Proof. Prove by contradiction. Suppose that $w_0 > 0$. Since there is an excess of producers in the frictionless matching market, $\overline{\Pi}_1 = \Pi_1$. So

$$\begin{aligned}\Pi_0 &= -w_0 + \delta\Big[\alpha_0\Pi_1 + (1-\alpha_0)(\beta\Pi_0 + (1-\beta)\overline{\Pi}_0)\Big] \\ &= -w_0 + \delta\Big[\alpha_0\overline{\Pi}_1 + (1-\alpha_0)(\beta\Pi_0 + (1-\beta)\overline{\Pi}_0)\Big].\end{aligned}$$

Also note that $\overline{\Pi}_0 = \delta\Big[\alpha_0\overline{\Pi}_1 + (1-\alpha_0)\overline{\Pi}_0\Big]$. Therefore,

$$\Pi_0 - \overline{\Pi}_0 = -w_0 + \delta(1-\alpha_0)\beta(\Pi_0 - \overline{\Pi}_0) < \delta(1-\alpha_0)\beta(\Pi_0 - \overline{\Pi}_0),$$

where the inequality comes from $w_0 > 0$. Since $\delta \in (0,1)$ and $\beta \in (0,1]$, we discuss whether $\alpha_0 = 1$. If $\alpha_0 < 1$, the equation above cannot be satisfied, which contradicts that $w_0 > 0$. On the other hand, if $\alpha_0 = 1$, $\overline{\Pi}_0 = \delta\overline{\Pi}_1$ and then $\Pi_0 = -w_0 + \delta\overline{\Pi}_1 < \overline{\Pi}_0$, which contradicts to (B-IC0) and also indicates that $w_0 = 0$. Therefore, $w_0 = 0$ and $\Pi_0 = \overline{\Pi}_0$. Thus both (B-IC1) and (B-IC0) bind.

We now show that (P-IC1) is slack. Suppose it binds, namely $U_1 = \overline{U}$. Then given $w_0 = 0$ and $\delta < 1$, plug in $U_1 = \overline{U}$ and get

$$\begin{aligned}U_0 &= \delta\Big[\alpha_0\overline{U} + (1-\alpha_0)(\beta U_0 + (1-\beta)\overline{U})\Big] \\ &< (1-\alpha_0)\beta U_0 + (1-(1-\alpha_0)\beta)\overline{U}.\end{aligned}$$

The inequality above indicates that $U_0 < \overline{U}$, which contradicts to (P-IC0). So (P-IC1) is slack. □

Lemma A.2. *For any buyer who directly contracts with producers, maximizing Π_1 is equivalent to minimizing w_1.*

Proof. Based on Lemma A.1, the buyer's continuation payoffs can be written as

$$\Pi_1 = \frac{1-\delta(1-\alpha_0)}{1-\delta\alpha_0\alpha_1 - \delta(2-\alpha_0-\alpha_1) + \delta^2(1-\alpha_0)(1-\alpha_1)}(y - w_1),$$

$$\Pi_0 = \frac{\delta\alpha_0}{1-\delta(1-\alpha_0)}\Pi_1.$$

Since $\frac{\partial \Pi_1}{\partial w_1} < 0$, a buyer's problem is equivalent to minimize w_1 subject to producer's incentive constraints. □

Proof of Lemma 1

For simplicity, we write w_1 as w in the rest of the proof. We complete the proof by analyzing and comparing the terms in the optimal bilateral relational contracts when choosing $\beta = 0$ or $\beta > 0$.

Choice 1: $\beta = 0$. If the optimal choice of β is 0, $U_1 = w - c + \delta\Big[(1-\alpha_1)U_1 + \alpha_1\overline{U}\Big]$. The buyer optimally chooses w subject to a binding (P-IC-e), which gives

$$w_{BS} = \frac{1}{\delta(1-\alpha_1)}c + (1-\delta)\overline{U}.$$

Choice 2: $\beta \in (0, 1]$. If the optimal choice of β is greater than 0, a buyer's optimization problem becomes $\min_{w,\beta} w$ subject to (P-IC-e), (P-IC0), (1), (2), $0 < \beta \le 1$, and $w \ge 0$.

The Lagrangian is given by

$$\begin{aligned} L = w &+ \lambda_1(U_1 - (w - c + \delta((1-\alpha_1)\cdot U_1 + \alpha_1(\beta U_0 + (1-\beta)\overline{U})))) \\ &+ \lambda_2(U_0 - \delta(\alpha_0 \cdot U_1 + (1-\alpha_0)\cdot(\beta U_0 + (1-\beta)\overline{U}))) \\ &+ \mu_1(w + \delta\overline{U} - U_1) + \mu_2(\overline{U} - U_0) + \mu_3(\beta - 1) + \mu_4(-w). \end{aligned}$$

The Kuhn-Tucker conditions are given by

$$\frac{\partial L}{\partial w} = 1 + \mu_1(1 - \frac{\partial U_1}{\partial w}) - \mu_2\frac{\partial U_0}{\partial w} - \mu_4 \le 0,$$

$$\frac{\partial L}{\partial w} w = 0,$$

$$\frac{\partial L}{\partial \beta} = -\mu_1 \frac{\partial U_1}{\partial \beta} - \mu_2 \frac{\partial U_0}{\partial \beta} - \mu_3 = 0,$$

$$\lambda_1, \lambda_2 > 0,$$

$$\mu_1, \mu_2, \mu_3, \mu_4 \geq 0,$$

$$\mu_1 (w + \delta \overline{U} - U_1) = 0,$$

$$\mu_2 (\overline{U} - U_0) = 0,$$

$$\mu_3 (1 - \beta) = 0,$$

$$\mu_4 w = 0.$$

Meanwhile, by taking derivatives on both sides of equations (1) and (2) with respect to w, get

$$\frac{\partial U_1}{\partial w} = 1 + \delta \Big[(1 - \alpha_1) \frac{\partial U_1}{\partial w} + \alpha_1 \beta \frac{\partial U_0}{\partial w} \Big], \tag{A1}$$

$$\frac{\partial U_0}{\partial w} = \delta \Big[\alpha_0 \frac{\partial U_1}{\partial w} + (1 - \alpha_0) \beta \frac{\partial U_0}{\partial w} \Big]. \tag{A2}$$

By taking derivatives on both sides of equations (1) and (2) with respect to β, get

$$\frac{\partial U_1}{\partial \beta} = \delta \Big[(1 - \alpha_1) \frac{\partial U_1}{\partial \beta} + \alpha_1 (U_0 - \overline{U} + \beta \frac{\partial U_0}{\partial \beta}) \Big], \tag{A3}$$

$$\frac{\partial U_0}{\partial \beta} = \delta \Big[\alpha_0 \frac{\partial U_1}{\partial \beta} + (1 - \alpha_0)(U_0 - \overline{U} + \beta \frac{\partial U_0}{\partial \beta}) \Big]. \tag{A4}$$

We proceed by the following steps.

Step 1: Show that $\mu_4 = 0$ and $w > 0$. Suppose that $w = 0$. Then

$$\begin{aligned} U_1 &= -c + \delta \Big[(1 - \alpha_1) U_1 + \alpha_1 (\beta U_0 + (1 - \beta) \overline{U}) \Big] \\ &< (1 - \alpha_1) U_1 + \alpha_1 (\beta U_0 + (1 - \beta) \overline{U}), \end{aligned}$$

where the inequality comes from that $c > 0$ and $\delta < 1$. The inequality above implies that

$$U_1 < \beta U_0 + (1-\beta)\overline{U} < U_0,$$

where the second inequality comes from (P-IC0). Meanwhile,

$$\begin{aligned} U_0 &= \delta\Big[\alpha_0 U_1 + (1-\alpha_0)(\beta U_0 + (1-\beta)\overline{U})\Big] \\ &< \alpha_0 U_1 + (1-\alpha_0)(\beta U_0 + (1-\beta)\overline{U}) \\ &< U_0, \end{aligned}$$

where the first inequality comes from $\delta < 1$ and the second inequality comes from (P-IC0) and $U_1 < U_0$. Since $U_0 < U_0$ can never be true, we know that $w > 0$ and thus $\mu_4 = 0$.

The implication for $w > 0$ is that

$$\frac{\partial L}{\partial w} = 1 + \mu_1(1 - \frac{\partial U_1}{\partial w}) - \mu_2 \frac{\partial U_0}{\partial w} = 0,$$

Step 2: Discuss the values of μ_1 and μ_2. **Case 1)** $\mu_1 = \mu_2 = 0$. In this case, $\frac{\partial L}{\partial w} = 0$ is violated, indicating that this case is not possible. In other words, at least one of two incentive compatibility constraints bind.

Case 2) $\mu_1 = 0$ and $\mu_2 > 0$. In this case, $U_1 > w + \delta\overline{U}$ and $U_0 = \overline{U}$. Solve U_1 and w based on equation (1) and get

$$U_1 = \frac{1-\delta(1-\alpha_0)}{\delta\alpha_0}\overline{U},$$

$$w = c + \frac{(1-\delta)^2 - \delta(1-\delta)(\alpha_0+\alpha_1)}{\delta\alpha_0}\overline{U}.$$

So $\frac{\partial U_1}{\partial \beta} = \frac{\partial U_0}{\partial \beta} = 0$. Meanwhile, $\frac{\partial L}{\partial w} = 0$ and $\mu_1 = 0$ imply that $1 = \mu_2 \frac{\partial U_0}{\partial w}$. $\frac{\partial L}{\partial \beta} = 0$ implies $\mu_3 = 0$, namely $\beta < 1$.

There are two conditions that need to be satisfied for this case to be feasible and optimal. First, for feasibility, the solved w and U_1 need to satisfy $U_1 > w + \delta\overline{U}$. After plugging in U_1 and

w as functions of $\overline{U}$, this requires that

$$\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_0}{1-\alpha_1}.$$

Second, since we are considering the case where choosing $\beta > 0$ is weakly better than choosing $\beta = 0$, the solved w needs to be lower than w_{BS}, which gives

$$\frac{(1-\delta)\overline{U}}{c} \leq \frac{\alpha_0}{1-\alpha_1}.$$

These two conditions contract each other, indicating that this case is not possible.

Case 3) $\mu_1 > 0$ and $\mu_2 > 0$. In this case, both incentive compatibility constraints bind, which give $U_1 = w + \delta\overline{U}$ and $U_0 = \overline{U}$. Under binding (P-IC-e) and (P-IC0), equations (1) and (2) can be satisfied only if

$$\frac{(1-\delta)\overline{U}}{c} = \frac{\alpha_0}{1-\alpha_1}.$$

In that case, the solved w also coincides with w_{BS}, and a buyer is thus indifferent among choosing any value of $\beta \in [0, 1]$.

Case 4) $\mu_1 > 0$ and $\mu_2 = 0$. In this case, $U_1 = w + \delta\overline{U}$, $U_0 > \overline{U}$, $\frac{\partial L}{\partial w} = 1 + \mu_1(1 - \frac{\partial U_1}{\partial w}) = 0$, and $\frac{\partial L}{\partial \beta} = -(\mu_1 \frac{\partial U_1}{\partial \beta} + \mu_3) = 0$.

We discuss whether $\beta = 1$ or not. If $\beta \neq 1$, $\mu_3 = 0$, then $\frac{\partial U_1}{\partial \beta} = 0$. Given that, equation (A3) suggests that $\frac{\partial U_0}{\partial \beta} < 0$, while equation (A4) suggests that $\frac{\partial U_0}{\partial \beta} > 0$. Therefore, a contradiction exists, indicating that $\beta = 1$.

Given $\beta = 1$ and $\mu_3 > 0$, solve

$$w_{BI} = \left(\frac{1}{\delta}\frac{1 + \frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1) + \frac{\delta}{1-\delta}\alpha_0}\right)c + \left(\frac{1 + \frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1) + \frac{\delta}{1-\delta}\alpha_0} - \delta\right)\overline{U}.$$

In sum, if a buyer decides to choose $\beta > 0$, she will optimally choose $\beta = 1$ with paying w_{BI} when her demand is 1. The comparison between choice 1 and choice 2 hinge on comparing w_{BS}

and w_{BI}. It turns out that $w_{BS} < w_{BI}$ if and only if

$$\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_0}{1-\alpha_1}.$$

Therefore, whenever the condition above holds, the buyer chooses $\beta = 0$ (separating from the producer when demand becomes 0) and pays w_{BS} when her demand is 1. Otherwise, she chooses $\beta = 1$ (retaining the producer when demand becomes 0) and pays w_{BI} when her demand is 1.

Proof of Lemma 2

Observe that the continuation payoffs and incentive compatibility constraints for an intermediary in an intermediated contract are similar with those for a producer in a bilateral contract. The only differences are that the "cost of production" for an intermediary is w_M and its continuation value after separating from a buyer, given by $\overline{V}$, is 0. Since

$$\frac{(1-\delta)\overline{V}}{w_M} \leq \frac{\alpha_0}{1-\alpha_1}$$

for any α_0 and α_1. Therefore, the value of p is determined by replacing c with w_M and $\overline{U}$ with 0 in w_{BI}.

Proof of Proposition 1

We prove the proposition in four steps.

Step 1: Compare bilateral contracting with separation with bilateral contracting with idleness. The following lemma comes directly from Lemma 1.

Lemma A.3. *If $\overline{U} > \overline{U}^{ei} \equiv \frac{c}{1-\delta}\alpha_0$, there exists a unique cutoff $\overline{\alpha}_1^{ei} \in (0,1)$ such that $w_{BS} < w_{BI}$ if and only if $\alpha_1 < \overline{\alpha}_1^{ei}$.*

Step 2: Compare bilateral contracting with separation with intermediated contracting.

Lemma A.4. *There exists a unique cutoff $\overline{\alpha}_1^{eo} \in (0,1]$ such that $w_{BS} < p$ if and only if $\alpha_1 < \overline{\alpha}_1^{eo}$.*

Proof. Observe that $w_{BS} < p$ if and only if

$$\frac{1}{\delta}\frac{1}{1-\alpha_1}c + (1-\delta)\overline{U} < (\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1)+\frac{\delta}{1-\delta}\alpha_0})\frac{c}{\delta} + (\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1)+\frac{\delta}{1-\delta}\alpha_0})(1-\delta)\overline{U},$$

or

$$\frac{1}{1-\alpha_1}\frac{c}{\delta} + (1-\delta)\overline{U} - \frac{1}{\delta - \frac{\delta}{1+\frac{\delta}{1-\delta}\alpha_0}\alpha_1}(\frac{c}{\delta} + (1-\delta)\overline{U}) < 0.$$

Let the LHS to be $f(\alpha_1) = \frac{1}{1-\alpha_1}\frac{c}{\delta} + (1-\delta)\overline{U} - \frac{1}{\delta - \frac{\delta}{1+\frac{\delta}{1-\delta}\alpha_0}\alpha_1}(\frac{c}{\delta} + (1-\delta)\overline{U})$. Observe that, when $\alpha_0 = 0$, $f(\alpha_1)\big|_{\alpha_0=0} = (1-1/\delta)\frac{c}{\delta}\frac{1}{1-\alpha_1} + (1 - \frac{1}{\delta(1-\alpha_1)})(1-\delta)\overline{U} < 0$. In this case, $w_{BS} < p$ for sure, so $\overline{\alpha}_1^{eo} = 1$.

When $\alpha_0 > 0$, observe that $f(0) = (1-\frac{1}{\delta})\kappa < 0$, and $f(1)$ goes to infinity. We now show that the continuous function $f(\alpha_1)$ intersects with 0 only once. Compute

$$\frac{\partial f(\alpha_1)}{\partial \alpha_1} = \frac{\zeta(\zeta\frac{c}{\delta} - \kappa)\alpha_1^2 - 2(c-\kappa)\zeta\alpha_1 + (\delta c - \zeta\kappa)}{(1-\alpha_1)^2(\delta - \zeta\alpha_1)^2}.$$

where $\zeta = \frac{\delta}{1+\frac{\delta}{1-\delta}\alpha_0}$ and $\kappa = \frac{c}{\delta} + (1-\delta)\overline{U}$.

Observe that the numerator is a quadratic equation, where the coefficient of α_1^2 is negative, the coefficient of α_1 is positive, and the constant $\delta c - \zeta(\frac{c}{\delta} + (1-\delta)\overline{U})$ can be positive or negative. Therefore, $f(\alpha_1)$ is either strictly increasing, or is first decreasing then increasing. In either case, $f(\alpha_1)$ intersects with 0, with the intersect being $\overline{\alpha}_1^{eo} \in (0,1)$.

In sum, there exists a unique cutoff $\overline{\alpha}_1^{eo} \in (0,1]$ such that $w_{BS} < p$ if and only if $\alpha_1 < \overline{\alpha}^{eo}$. $\square$

Step 3: Compare bilateral contracting with idleness with intermediated contracting.

Lemma A.5. *If $\overline{U} > \overline{U}^{io} \equiv \frac{(1-\delta(1-\alpha_0))c}{\delta(\delta(2-(1-\delta)\alpha_0)-1)}$, there exists a unique cutoff $\overline{\alpha}_1^{io} \in (0,1)$ such that $w_{BI} < p$ if and only if $\alpha_1 < \overline{\alpha}_1^{io}$. Otherwise, $w_{BI} < p$ for sure.*

Proof. Observe that $w_{BI} < p$ if and only if

$$(\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1)+\frac{\delta}{1-\delta}\alpha_0})c + (\frac{1+\frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1)+\frac{\delta}{1-\delta}\alpha_0} - \delta)\overline{U} < (\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_0}{(1-\alpha_1)+\frac{\delta}{1-\delta}\alpha_0})(\frac{c}{\delta} + (1-\delta)\overline{U}),$$

or

$$\frac{1+\frac{\delta}{1-\delta}\alpha_0}{1+\frac{\delta}{1-\delta}\alpha_0-\alpha_1}\left(\frac{2\delta-1}{\delta}\overline{U}-\frac{1-\delta}{\delta}\frac{c}{\delta}\right)<\delta\overline{U}.$$

Let the LHS to be $g(\alpha_1)=\frac{1+\frac{\delta}{1-\delta}\alpha_0}{1+\frac{\delta}{1-\delta}\alpha_0-\alpha_1}(\frac{2\delta-1}{\delta}\overline{U}-\frac{1-\delta}{\delta}\frac{c}{\delta})$. Observe that $g(0)=\frac{2\delta-1}{\delta}\overline{U}-\frac{1-\delta}{\delta}\frac{c}{\delta}$ and $g(1)=\frac{1+\frac{\delta}{1-\delta}\alpha_0}{\frac{\delta}{1-\delta}\alpha_0}(\frac{2\delta-1}{\delta}\overline{U}-\frac{1-\delta}{\delta}\frac{c}{\delta})$. Also observe that $g(0)-\delta\overline{U}=\frac{-(1-\delta)^2}{\delta}\overline{U}-\frac{1-\delta}{\delta}\frac{c}{\delta}<0$.

If $\overline{U}<\overline{U}^{io}$, $g(1)<\delta\overline{U}$ and thus the inequality holds for sure. Otherwise, since $g(\alpha_1)$ is strictly increasing in α_1, there exists a unique cutoff $\overline{\alpha}_1^{io}$ by the intermediate value theorem. □

Step 4: Let $\overline{U}^*=\overline{U}^{io}$ and $\alpha^*=\min\{\overline{\alpha}_1^{io},\overline{\alpha}_1^{eo}\}$.

If $\overline{U}<\overline{U}^{io}$, $w_{BI}<p$ by Lemma A.5. Otherwise, if $\overline{U}\geq\overline{U}^{io}$ there are two cases. If $w_{BS}<w_{BI}$, intermediated contract is optimal when $p<w_{BS}$, namely when $\alpha_1>\overline{\alpha}_1^{eo}$. If $w_{BS}>w_{BI}$ intermediated contract is optimal when $p<w_{BI}$, namely when $\alpha_1>\overline{\alpha}_1^{io}$.

Proof of Corollary 1

First, note that $w_B(\alpha_i)>w_M$ for all $i\in\mathcal{I}_{BS}\cup\mathcal{I}_{BI}$. Therefore, $E[w_B(\alpha_i)\mid i\in\mathcal{I}_{BS}\cup\mathcal{I}_{BI}]>E[w_M|i\in\mathcal{I}_M]$. Second, note that $w_B(\alpha_i)$ takes on different values depending on α_i, which is heterogeneous across buyers, while w_M is constant. Therefore, $\mathrm{Var}[w_B(\alpha_i)\mid i\in\mathcal{I}_{BS}\cup\mathcal{I}_{BI}]>\mathrm{Var}[w_M\mid i\in\mathcal{I}_M]=0$. Third, the separation rate is given by $\beta_i\alpha_{1i}$. Note that $\beta_i=1$ for all $i\in\mathcal{I}_{BI}$, while $\beta_i=0$ for all $i\in\mathcal{I}_{BS}\cup\mathcal{I}_M$. Fourth, the idleness for a buyer $i\in\mathcal{I}_{BI}$ is given by $(1-\pi_i)(1-\omega_i)>0$. By contrast, for $i\in\mathcal{I}_{BS}\cup\mathcal{I}_M$, idleness is zero.

Proof of Corollary 2

With the introduction of intermediaries, the per-period payoff of buyer $i\in\mathcal{S}_0$ increases by $y-p(\alpha_i)$. The per-period payoff of buyer $i\in\mathcal{S}_I$ increases by $w_{BI}(\alpha_i)-p(\alpha_i)$. The per-period payoff of buyer $i\in\mathcal{S}_S$ increases by $w_{BS}(\alpha_i)-p(\alpha_i)$. The per-period payoff of the remaining buyers are unchanged.

The average producer pay per unit effort falls, since $w_B(\alpha_i)>w_M$ for all $i\in\mathcal{S}_I\cup\mathcal{S}_S$. Producer separation rates and idleness also fall, since buyers switch from bilateral contracts with either idleness and separation to intermediated contracts in which buyers are never idle and never separate.

The measure of buyers who receive services increases by $|\mathcal{S}_0|$. The total measure of services provided in the economy increases by $\int_{\mathcal{S}_0} \pi_i dF$. However, for $i \in \mathcal{S}_I$, the steady-state measure of producers matched to these buyers fall, since idleness falls. The total measure of matched producer $(n_M + n_B)$ therefore increases if and only if $\int_{\mathcal{S}_0} \pi_i dF > \int_{\mathcal{S}_I} (1 - \pi_i) dF$.

Proof of Corollary 3

Based on Lemma 1 and 2, the payments by buyer i under bilateral contracts or intermediation are

$$w_B(\alpha_i) = \frac{1}{\delta} \frac{1}{1 - \alpha_i} c + (1 - \delta)\overline{U},$$

$$w_G = w_M = \frac{c}{\delta} + (1 - \delta)\overline{U},$$

$$p(\alpha_i) = \frac{1}{\delta} \frac{1 + \frac{\delta}{1-\delta}\alpha_i}{1 + \frac{\delta}{1-\delta}\alpha_i - \alpha_i} \left[\frac{c}{\delta} + (1 - \delta)\overline{U}\right],$$

where $w_B(\alpha_i)$ is the pay for a specialist, w_G is the pay for a generalist, w_M is the pay from an intermediary to an intermediated producer, and $p(\alpha_i)$ is the service fee for intermediation.

The buyer's post-matching continuation payoffs when she has demand when choosing to directly contract with specialists, to directly contract with a generalist, or to outsource are thus

$$\Pi_B(\alpha_i) = \frac{y + \Delta_i}{1 - \delta} - \left[\overline{U} + \frac{c}{(1 - \alpha_i)\delta(1 - \delta)}\right],$$

$$\Pi_G = \frac{y}{1 - \delta} - \left[\overline{U} + \frac{c}{\delta(1 - \delta)}\right].$$

$$\Pi_M(\alpha_i) = \frac{y + \Delta_i}{1 - \delta} - \left[\frac{1}{\delta} \frac{1 + \frac{\delta}{1-\delta}\alpha_i}{1 + \frac{\delta}{1-\delta}\alpha_i - \alpha_i} (\overline{U} + \frac{c}{\delta(1 - \delta)})\right],$$

respectively.

Three observations follow. First, a buyer prefers to bilaterally contract with a specialist than bilaterally contract with a generalist if and only if $\Pi_B(\alpha_i) \geq \Pi_G$, or,

$$\Delta_i \geq \frac{\alpha_i}{1-\alpha_i}\frac{c}{\delta}.$$

Second, a buyer prefers to bilaterally contract with specialists than intermediate if and only if

$$\left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{1-\delta}\alpha_i-\alpha_i}-1\right]\left[\overline{U}+\frac{c}{\delta(1-\delta)}\right] \geq \frac{\alpha_i}{1-\alpha_i}\frac{c}{\delta(1-\delta)}.$$

By Lemma A.3, there exists a unique cutoff α^{ei} such that the buyer prefers to intermediate if and only if $\alpha_i > \alpha^{ei}$. Third, a buyer prefers an intermediated contract over bilateral contract with a generalist if and only if

$$\Delta_i \geq \overline{\Delta}(\alpha_i) \equiv \left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{1-\delta}\alpha_i-\alpha_i}-1\right]\left[\overline{U}+\frac{c}{\delta(1-\delta)}\right].$$

Therefore, the buyer choose to intermediate if and only if α_i and Δ_i are both large enough.

Proof of Corollary 4

Let $\mathcal{S}_G$ denote buyers who switch from bilateral contracts with generalists to intermediated contracts with specialists when intermediaries become available. Let $\mathcal{S}_B$ be the set of buyers who switch from bilateral contracts with specialists to intermediated contracts with specialists. Given the assumption on the distribution of (Δ_i, α_i), $|\mathcal{S}_G|, |\mathcal{S}_B| > 0$. The contractual choice of the remaining buyers are unchanged. Therefore, the measure of specialists unambiguously increase with intermediation.

Note that the Bellman equation for unmatched producers can be rewritten as

$$\overline{U} = \frac{v}{u}\frac{E[v_i U_{1i}]}{v} + \left(1-\frac{v}{u}\right)\delta\overline{U}.$$

With the introduction of intermediaries, more producers are contracted as specialists under intermediated contracts without separation, so v and $E[v_i U_{1i}]$ both fall. This implies that $\frac{v}{u}$

increases. It follows that the measure of unmatched producers $n_N = u - v$ falls.

Proof of Corollary 5

In a two-task economy with reputable intermediaries, the service fee is given by

$$p(\alpha_i) = \left(\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{(1-\alpha_i)+\frac{\delta}{1-\delta}\alpha_i}\right)w_M + \left(\frac{1+\frac{\delta}{1-\delta}\alpha_i}{(1-\alpha_i)+\frac{\delta}{1-\delta}\alpha_i} - \delta\right)(-\gamma\tilde{V})$$

This implies that

$$\Pi_M(\alpha_i) = \frac{y+\Delta_i}{1-\delta} - \left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{1-\delta}\alpha_i - \alpha_i}(\overline{U} + \frac{c}{\delta(1-\delta)})\right] + \left(\frac{1+\frac{\delta}{1-\delta}\alpha_i}{(1-\alpha_i)+\frac{\delta}{1-\delta}\alpha_i} - \delta\right)\frac{\gamma\tilde{V}}{1-\delta},$$

As γ increases, $\Pi_M(\alpha_i)$ rises, so buyers switch to intermediated contracts with specialists from bilateral contracts with both generalists and specialists. Therefore, by the same logic as Corollary 4, the measure of specialists increases, while the measure of unmatched producers decreases.