# The Emergence of Managers in Relational Contracting Markets

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July 2025

#### Abstract

Managers who enforce norms and allocate resources pervade economic exchange. We develop repeated-game models to explain when and why managers emerge in relational contracting environments. In our models, managers mitigate a fundamental tension between incentive provision and flexible allocations, but require incentive rents, leading to endogenous sorting into organizational arrangements. Agents choose managerial coordination over decentralized exchange when their idiosyncratic demand is volatile and specialized services are needed. Managerial coordination also increases as communication frictions decline. The models provide novel microfoundations for the structure, boundaries, and distributional impacts of organizations such as professional service firms, global sourcing firms, and online platforms.

Keywords: managerial coordination, relational contracts JEL: D23, L22, L24

<sup>\*</sup>First version: March 2024. Li: Cornerstone Research (duoxili@bu.edu); Wong: University of Hong Kong (mbwong@hku.hk). This paper previously circulated as "The Manager as a Nexus of Relational Contracts." We thank Dan Barron, Lisa Bernstein, Eric Budish, Anthony Casey, Wouter Dessein, Florian Englmaier, Matthias Fahn, Luis Garicano, Bob Gibbons, Leshui He, Jin Li, Marta Troya-Martinez, Dilip Mookherjee, Andy Newman, Juan Ortner, Joel Watson, Birger Wernerfelt, Giorgio Zanarone, and seminar participants at HKU, MIT, Hong Kong Economic Association, SIOE, and the 9th Workshop on Relational Contracts for helpful comments and suggestions.

# **1** Introduction

Managers, brokers, and institutional leaders play pivotal roles in economic exchange by enforcing norms and coordinating allocations as centralized nodes in relational networks. Their prominence underscores the fact that *relational contracts*—informal agreements sustained by repeated interaction—are pervasive in labor markets, supply chains, and other economic institutions.<sup>1</sup> Game-theoretic models have formalized how relational contracts operate, demonstrating how reciprocity and dynamic incentives enable cooperation (e.g., Levin 2003; Halac 2012; Watson, Miller and Olsen 2020). Recent advances extend these frameworks to decentralized matching markets, revealing how relational contracts form when agents freely meet (Board and Meyer-ter-Vehn 2015; Fahn and Murooka 2025).<sup>2</sup>

Yet a central puzzle endures: Why do *centralized* relational structures—such as managerial hierarchies or intermediation networks—emerge to allocate resources, even where decentralized exchange is feasible? Resolving this question is critical for both theory and practice. Centralized relational structures are ubiquitous in real-world markets, implying inherent efficiency advantages. Yet formal models explaining their prevalence remain underdeveloped. Bridging this gap would not only advance the theory of relational contracts, but also clarify the organizational logic underpinning key economic institutions.

In this paper, we analyze a simple general equilibrium model of an exchange economy with imperfect contractual enforcement, in which resource-allocating managers endogenously emerge as centralized nodes in relational contracting markets. The model clarifies why it is efficient when trust is needed that managers partially replace competitive matching markets in coordinating allocations. We characterize the individual- and market-level determinants of each market participant's choice between managerial coordination and decentralized exchange. We show how managers affect economic output, the division of labor, matching market flows, and the distribution of rents. We then use the model as a lens to understand

<sup>&</sup>lt;sup>1</sup>The ubiquity of relational contracts is documented by a large body of research spanning anthropology (Geertz 1962, 1978), sociology (Macaulay 1963), law (Macneil 1978), and political science (Ostrom 1990).

<sup>&</sup>lt;sup>2</sup>For surveys of recent work on relational contracts in economics, see MacLeod (2007), Malcomson (2013), Gil and Zanarone (2017), Macchiavello and Morjaria (2023), and Muehlheusser, Fahn and MacLeod (2023).

recent structural shifts in economic organization.

Why does the *visible hand* of managers supplant the *invisible hand* of markets? In our model, managers emerge to alleviate a conflict between incentive provision and flexible assignment that arises in fluctuating environments. The precise mechanism is formalized using a variant of MacLeod and Malcomson (1989). Following prior models that study environments with imperfect contractual enforcement, we assume that buyers cannot compel producers to exert effort using formal contracts, so buyers must motivate producers using future surplus in relational contracts. Buyers and producers meet and enter contracts in frictionless matching markets without search delays or hidden payoff-relevant information, as in Board and Meyer-ter-Vehn (2015). Departing from prior models, we introduce a new feature—fluctuating buyer demand—to capture a realistic and relevant aspect of repeated business interactions (Macchiavello and Morjaria 2015). We also introduce managers who cannot themselves produce, but can manage—i.e., monitor and allocate—a large number of producers. In equilibrium, managers endogenously enter multiple relational contracts on both sides of the market, aggregate fluctuating demand, coordinate assignments, and keep track of producer performance.

The model reveals that the use of managers involves a trade-off between demand aggregation and double marginalization. The benefit of managers is that they aggregate fluctuating buyer demand and thereby prevent producers from becoming idle or unmatched. Therefore, managers can promise longer and more productive relational contracts and thereby reduce the incentive rent needed to motivate producers to exert effort. However, managers must themselves be motivated by an additional incentive rent. The cost of managers is thus a form of double marginalization.<sup>3</sup>

Because of this trade-off, it is found that buyers sort into managerial coordination

<sup>&</sup>lt;sup>3</sup>In the classic literature in industrial organization, double marginalization occurs when both upstream and downstream parties possess market power and use restrictive linear contracts. This form of double marginalization can be eliminated by vertical integration of pricing decision rights, resale-price maintenance, or non-linear pricing (Tirole 1988). In our model, double marginalization instead arises from the non-verifiability of contract performance and the non-transferability of production and monitoring capabilities, as in models of middleman margins (e.g., Biglaiser and Friedman 1994; Bardhan, Mookherjee and Tsumagari 2013). Conventional remedies do not eliminate this form of double marginalization.



Figure 1: Endogenous relational contracting structures in steady-state equilibrium

Demand volatility

and decentralized exchange. Their optimal choice depends on individual characteristics, such as their individual demand volatility. It also depends on market-level characteristics, such as market tightness and market size. If the matching market is sufficiently tight, managers emerge as centralized contracting nexuses. Buyers with short-lived demand contract with managers. Buyers with long-lived demand contract directly with producers in a sea of decentralized pairings. Figure 1 illustrates the relational contracting structures that endogenously emerge in the unique steady-state equilibrium of the exchange economy.

Several comparative statics are derived regarding the determinants and impacts of managerial coordination. First, when compared to managed producers, directly contracted producers have higher average incentive pay, more dispersed pay, higher separation rates, and higher idleness. Second, the introduction of managers into an economy benefits buyers with fluctuating demand, but reduces the pay of producers initially earning pay premiums. As such, managerial coordination is both efficiency-enhancing and redistributive.

Two extensions analyze the relationships between managerial coordination, specialization, and the strength of market-level reputation effects. We show that managed producers are more specialized and that managerial coordination increases producer specialization. Moreover, buyers are more likely to choose managerial coordination if there are gains from producer specialization and if the managers have reputation concerns arising from word-of-mouth communication about manager performance. We also show that, unlike intermediation that mitigates search frictions or adverse selection, managerial coordination may *increase* as communication frictions fall. The reason is that ease of communication may allow buyers to better coordinate on collective punishments against managers who shirk. This lowers manager incentive rents, so managerial coordination becomes more attractive.

Our theoretical results provide a novel lens for understanding recent transformations in social and economic structure. In the past few decades, novel organizational forms—each characterized by centralized resource allocation—have increasingly displaced traditional decentralized exchange. Labor markets, for example, have been fundamentally altered by professional service firms that monitor and deploy large numbers of workers (e.g., legal, accounting, IT, and HR firms).<sup>4</sup> A parallel revolution has reshaped international trade, where global sourcing firms increasingly manage vast supplier networks and coordinate significant shares of world production.<sup>5</sup> More recently, consumer markets have been radically restructured by digital platforms like Uber and Airbnb, which allocate and monitor increasingly large numbers of independent contractors.

Connections between our model and the empirical literature on professional service firms are discussed throughout the paper. As we shall show, the model generates realistic predictions not only for the drivers of professional service outsourcing, but also for the effects of professional service firms on both workers and the labor market as a whole.

The rest of the paper proceeds as follows. Section 2 discusses related literature. Section 3 introduces our model. Section 4 characterizes and compares direct and managed contracts, taking matching market conditions as given. Section 5 derives the steady-state equilibrium by endogenizing matching market flows. Section 6 pursues extensions. Section 7 concludes.

<sup>&</sup>lt;sup>4</sup>As of 2024, professional and business service firms accounted for 13% of U.S. employment. See Weil (2014).

<sup>&</sup>lt;sup>5</sup>Ahn, Khandelwal and Wei (2011) estimate that intermediaries facilitated 20% of China's exports in 2005, while Bernard, Grazzi and Tomasi (2015) find that over 25% of Italian exporters act as intermediaries, accounting for 10% of national exports. See also Belavina and Girotra (2012) and Antràs and Chor (2022).

# 2 Related Literature

The contribution of this paper is to formally demonstrate how centralized resource allocation endogenously emerges in relational contracting markets. In doing so, we derive a rich set of predictions regarding the drivers of managerial coordination, as well as its impact on output, the division of labor, matching market flows, and the distribution of rents. The contribution not only advances the literature on relational contracts, but also the literatures on managers, intermediaries, firm boundaries, and market institutions.

In the literature on relational contracts, our work builds on repeated-game models of relational contracts in decentralized matching markets (Shapiro and Stiglitz 1984; MacLeod and Malcomson 1989, 1998; Yang 2008; Board and Meyer-ter-Vehn 2015; Fahn 2017; Powell 2019; Li 2022; Fahn and Murooka 2025). To these models, we introduce fluctuating demand and managers with neither intrinsic demand nor productive capability. The managers aggregate fluctuating demand and coordinate assignments, easing a tension between incentive provision and flexible allocations, and thereby enabling trade and specialization. Previously, Board (2011) showed that the same tension can lead buyers to restrict their number of trading partners. Andrews and Barron (2016) showed that it can cause dynamic allocations to depend on payoff-irrelevant past performance. Li and Powell (2020) highlighted that a similar tension can lead agents to interact in multiple activities. To our knowledge, this paper is the first to show how the same tension leads to centralized resource allocation.

In the literature on managers, a large body of work has underscored that managers (1) monitor performance, (2) allocate talent to tasks, and (3) enforce relational contracts (Bandiera, Barankay and Rasul 2007; Gibbons and Henderson 2012; Minni 2023; Caplin et al. 2023). However, existing general equilibrium models of managers abstract entirely from the incentive problems that many scholars believe managers exist to remedy. Lucas (1978) provided a model of occupational choice in which managerial talent is scarce and highly complementary to production workers, leading to high pay for managers. Garicano (2000) pioneered a model of knowledge hierarchies, in which managers function as expert problem-solvers that can help less skilled workers. While these models of managers generate

rich predictions regarding efficiency, inequality, and organizational structure, the model presented here is the first to do so with incentive-based microeconomic foundations.

In the literature on intermediaries, many models study middlemen who overcome search frictions (e.g. Rubinstein and Wolinsky 1987). Another set studies intermediaries who overcome adverse selection (e.g. Biglaiser 1993). However, it is a puzzle for these theories why the advent of the Internet has not led to radical disintermediation, but has instead spurred the growth of intermediary firms (Belavina and Girotra 2012; Bergeaud et al. Forthcoming). We instead study intermediaries that purely overcome moral hazard. We show how a reduction in communication frictions can *increase* the extent of intermediation.

In the literature on firm boundaries, many economic forces have been cataloged as determinants of firm boundaries since the seminal work of Coase (1937) and Williamson (1971, 1975, 1985). A prominent strand of the formal literature studies how firm boundaries are shaped by the optimal allocation of asset ownership (Grossman and Hart 1986; Gibbons 2005). Another explores the differences between service and employment contracts, emphasizing differences in bargaining and contract-writing costs (Wernerfelt 1997, 2015, 2016; Hart and Moore 2008; Levin and Tadelis 2010; Tadelis and Williamson 2013), or differences between spot and relational contracting (Simon 1951; Bolton and Rajan 2003; Raith 2022). Our paper takes a different but complementary approach. We suggest that the labor boundaries of firms are shaped by market participants' choices between direct and managed relational contracts. Our approach can be combined with previous approaches to jointly understand the ownership, contracting, and management structure of firms.<sup>6</sup>

The model developed here can be viewed as a formalization of informal intuitions from early contract-based theories of the firm. Alchian and Demsetz (1972) proposed that the essence of the firm is the presence of a central contracting party that monitors performance and excludes underperforming team members from future participation. Jensen and Meckling (1976, pp. 310–311) expanded on this view and suggested that "contractual relations are the essence of the firm, not only with employees but with suppliers, customers, creditors, and

<sup>&</sup>lt;sup>6</sup>For instance, Baker, Gibbons and Murphy (2002) provide models in which asset ownership alters the strength of relational contracts.

so on." Demsetz (1988, pp. 154) concurred that "the firm properly viewed is a 'nexus' of contracts" and elaborated that the key attributes of firm-like organization are continuity of association, specialization, and managerial direction. Our micro-founded model shows how resource-allocating managers endogenously emerge as nexuses of *relational* contracts.

In formalizing these intuitions, our model extends and clarifies related ideas in the literatures on supply chain management and market institutions. Belavina and Girotra (2012) developed a model of relational intermediaries in supply chains with two buyers, two sellers, and non-transferable utilities. Our transferable utility model is similar but enables a fuller characterization of the determinants and welfare consequences of intermediation. Milgrom, North and Weingast (1990) used a model of repeated Prisoner's Dilemma games to show that private judges in medieval trade can create trust with less observability than reputation systems.<sup>7</sup> This paper follows a similar logic. However, our formal approach is different, and our focus is not the historical role of legal and regulatory institutions. In our view, the problem of trust remains pervasive despite the development of modern institutions. The need for trust explains the ubiquity of managerial coordination.

# 3 Model

**Basics.** Time is discrete and infinite,  $t \in \{0, 1, ...\}$ . There is a unit mass of heterogeneous buyers indexed by *i*. There is a measure *n* of identical producers. There is a finite number *K* of identical managers. All players are infinitely-lived and have a common discount factor  $\delta \in (0, 1)$ . The measure of producers, *n*, is determined by endogenous entry subject to entry cost *C* > 0. Entry cost *C* represents the producers' training or opportunity cost.

**Demand realization.** At the start of each period, the demand of each buyer *i*, denoted as  $d_{it} \in \{0, 1\}$ , is realized and publicly observed.<sup>8</sup> Each buyer's demand  $d_{it}$  is independently

<sup>&</sup>lt;sup>7</sup>Related papers on market institutions include Greif (1993, 2006), Greif, Milgrom and Weingast (1994), Calvert (1995), and Fafchamps (2004).

<sup>&</sup>lt;sup>8</sup>To focus on the relational incentives, we abstract from any adverse selection problem in the model. Therefore, when contracting with buyers, producers know whom they are dealing with and have the correct

drawn following a Markov process. The demand-switching probabilities are each buyer's publicly known type  $\alpha_i = (\alpha_{1i}, \alpha_{0i})$ : With probability  $\alpha_{1i}$ , buyer *i*'s demand switches from 1 to 0. With probability  $\alpha_{0i}$ , buyer *i*'s demand switches back from 0 to 1. The buyers types have distribution *F* on  $[0, 1] \times [0, 1]$ .

**Matching.** After demand is realized, buyers may offer contracts to either producers or managers. Since managers cannot produce by themselves, managers fulfill buyer contract requirements by in turn contracting with producers. The timing is as follows: (1) Buyers offer contracts to managers and producers, (2) managers are matched with buyers, (3) managers offer contracts to producers, and (4) producers are matched with either buyers or managers. Each contract is a contingency plan specifying compensation, effort levels, and the probability of continuation.

Following Board and Meyer-ter-Vehn (2015), we model the matching of producers using two axioms. First, it is assumed to be *frictionless*, meaning that all offers are accepted subject to participation constraints. Second, it is assumed to be *anonymous*, meaning that each producer's probability of receiving a contract offer does not depend on their past behavior. The assumption of anonymous matching for producers is standard and greatly simplifies analysis. In addition, we assume that matching is *random*, so unmatched producers have the same probability of becoming matched. Matched producers do not receive offers.<sup>9</sup>

The matching process between buyers and managers is assumed to be frictionless, random, but *not* anonymous. If a manager violates its contract and becomes unmatched from a buyer, the buyer can refuse to contract with the manager forever. The anonymity assumptions are important for our results and will be discussed in Section 3.2.

**Direct contracts.** If a buyer matches with a producer, the remainder of the period proceeds as follows. First, the buyer makes a payment  $w_t \ge 0$  to the producer. The producer then

expectation of how the relationships will go.

<sup>&</sup>lt;sup>9</sup>Board and Meyer-ter-Vehn (2015) analyzes relational contracts in a frictionless matching market where matched producers may receive on-the-job offers. They show that this leads to heterogeneous productivity across otherwise identical matches. We rule this possibility out for simplicity.



Figure 2: Structure of direct and managed contracts

chooses an effort level, denoted by  $e_t \in \{0, 1\}$ . The cost of effort is given by  $c(e_t)$ , where c(0) = 0 and c(1) = c > 0. The effort generates an output for the buyer only if demand is positive, so  $y_{it} = yd_{it}e_t$ . Effort and output are observable by the buyer but are not verifiable by a court.

**Managed contracts.** If a buyer matches with a manager, she pays a service fee  $p_t \ge 0$  to the manager. The manager chooses to assign one or none of its producers to the buyer. The manager pays  $w_t$  to the producer. The producer then exerts costly effort  $e_t$  and produces output  $y_{it}$  for the buyer. Effort and output are observable by both the buyer and the manager, but are not verifiable by a court.

**Separation.** Either party in a match can choose to terminate their contract and separate from each other both after demand realization and after production. If separation occurs after demand realization, both parties can participate in the producer or manager market matching in the current period. However, if separation occurs after production, they need to wait till the next period to rematch.

**Payoffs.** In a direct contract, the buyer's payoff is  $y_{it} - w_t$  in each period t. In a managed contract, the buyer's payoff is  $y_{it} - p_t$ . The manager's payoff per service demand is  $p_t - w_t$ . The producer's payoff is  $w_t - c(e_t)$ .



Figure 3: Timeline of one period

At the start of the period, buyer demand is realized and publicly observed, after which separations may occur. Unmatched buyers can then match with either producers or managers. Managers can also match with producers. In a direct contract, the buyer pays the producer and the producer exerts effort. In a managed contract, the buyer pays the manager, the manager assigns a producer to the buyer and pays the producer, and the producer then exerts effort. Separation may occur at the end of the period.

**Remark.** This model has two key features that are not present in standard models of relational contracting in frictionless matching markets, such as Shapiro and Stiglitz (1984). First, we allow buyers to have fluctuating demand. Second, we introduce managers who can enter relational contracts with a multitude of agents on both sides of the market. These managers do not have intrinsic demand or productive ability. What they can do, however, is to match with a large set of buyers and producers and monitor the performance of all of their matched producers. As we shall show, when demand is volatile and the cost of idleness is high, managers can sustain cheaper relational contracts on both sides of the market by reassigning their producers across buyers. Buyers may therefore prefer managerial coordination to direct contracting.

## **3.1 Empirical Applications**

This subsection shows how the model can be used to think about professional service firms, global sourcing firms, and online platforms.

**Example 1** (Professional service firms). Consider an entrepreneur (*buyer*) who needs a service performed. She can fulfill the demand either by employing an in-house worker (*producer*) or contracting it out to a professional service firm (*manager*) that allocates its employees to clients. Entrepreneurs may make these employ-or-outsource decisions for various professional services, including cleaning, security, accounting, legal, IT, HR, and

temporary help services. These decisions determine the labor boundary of firms. We can define *employment* as a direct relational contract between an employer and a worker. We then define *outsourcing* as a managed contract in which an professional service firm employs workers and assigns them to different entrepreneurs. Under these definitions, the model can be used to predict the determinants and effects of professional service outsourcing.

**Example 2** (Global sourcing firms). Consider a retailer (*buyer*) who needs a manufacturing service performed. It can fulfill the demand either by directly contracting with a manufacturer (*producer*) or contracting through a global sourcing firm (*manager*) such as Li & Fung or Shein. Such firms maintain a large number of relationships with upstream manufacturers and function as nexuses of relational contracts (Belavina and Girotra 2012). We can define the service contract between the retailer and the manufacturer as a direct contract and that between the retailer and the sourcing firm as a managed contract. Under these definitions, the model can be used to analyze the scope and effects of global sourcing firms.

**Example 3** (Online platforms). Consider a consumer (*buyer*) who needs a service performed. It can fulfill the demand either by directly contracting with a service provider (*producer*) or contracting through an online platform (*manager*). Under these definitions, the model can be used to analyze online platforms that function as monitor-allocators. Examples of such platforms include Uber or Airbnb, who maintain relationships on both sides of the market, track provider performance, and direct consumers to high-performing providers.<sup>10</sup>

# **3.2 Discussion of Key Assumptions**

This subsection discusses key assumptions. First, the number of managers is assumed to be finite. This assumption is crucial for the demand aggregation mechanism studied in the model. Because of the assumption, each manager matches with a continuum of buyers with fluctuating demand. Managers are therefore guaranteed a constant volume of buyer demand

<sup>&</sup>lt;sup>10</sup>Here we think of the function of the platform's algorithmic computer systems as similar to the allocative and monitoring function traditionally performed by human managers in professional service firms.

by the law of large numbers. By flexibly assigning matched buyers to matched producers, managers can then offer relational contracts with constant demand to producers.

Second, matching between buyers and managers is assumed to not be anonymous. This assumption is important for our results. If the matching is anonymous instead, a buyer who separates from a manager will immediately rematch with the same manager with positive probability, but the buyer will not know the identity or history of this manager. A larger incentive rent will therefore be needed to motivate the manager. By assuming that matching between buyers and managers is not anonymous, the incentive rent needed to motivate the manager is lowered, so managed contracts become more attractive relative to direct contracts.

Third, matching between managers and producers is instead assumed to be anonymous. This assumption is standard but admittedly unnatural. It implies that a producer who separates from a manager will immediately rematch with the same manager with positive probability, and the manager will not know the identity or history of this producer. If the anonymity assumption is relaxed, then managers can offer contracts with lower incentive rents to producers than buyers can simply because managers receive a larger volume of demand (a la Biglaiser and Friedman 1994). We deliberately omit this channel not only for simplicity, but also to highlight that managers can emerge even in its absence. In our model, managers emerge purely to overcome the tension between incentive provision and flexible allocations.<sup>11</sup>

Fourth, payments  $w_t$  and  $p_t$  are assumed to be nonnegative. This *limited liability* assumption is proved to be without loss of generality in Online Appendix A.<sup>12</sup> It is made for two reasons. First, it is realistic in many real-world settings. Second, it significantly simplifies the analysis. Because of limited liability, equilibrium pay is always zero during periods when demand is zero (see Lemma A.1). One can thus characterize each buyer's

<sup>&</sup>lt;sup>11</sup>Biglaiser and Friedman (1994) show middlemen with reputations have a larger incentive to monitor and are in a better position to learn about quality than an ordinary buyer does because they buy a larger proportion of the producers' goods (see also Bardhan, Mookherjee and Tsumagari 2013). In contrast, managers do not have reputations in our baseline model; they simply enter relational contracts on both sides of the market.

<sup>&</sup>lt;sup>12</sup>Specifically, we show that if limited liability constraints are violated in an equilibrium relational contract, either there exists another equilibrium relational contract where the limited liability constraints hold, or the original relational contract cannot be sustained.

choice between stationary direct and managed contracts simply by comparing the buyer's per-period payment when demand is one.

Fifth, ex-post bonus payments are disallowed. This assumption is proved to be without loss of generality in Online Appendix A. The reasoning is as follows. There is an excess of producers, so buyers in our model have all the bargaining power. Therefore, for any contract with bonuses, there is a weakly more profitable contract for buyers without bonuses. Intuitively, the buyer prefers to pay the producer at the latest possible moment before  $e_t$ . It is therefore weakly profitable to shift the bonus  $b_{t-1}$  into the next period's expected pay, such that it is paid only when the next period's demand is one and the producer is retained. Since effort is perfectly observable, the proof for a similar result in Board and Meyer-ter-Vehn (2015) can be extended to our setting despite the presence of limited liability.<sup>13</sup>

Finally, the measure of producers n is assumed to determined by endogenous entry. This assumption makes the supply of producers perfectly elastic, thus simplifying the equilibrium analysis. The case where n is exogenous is analyzed in Online Appendix B.

# 4 Direct and Managed Relational Contracts

In this section we characterize optimal direct and managed relational contracts. We then analyze the buyer's choice between direct and managed contracts. This analysis explains why and when managed contracting may be preferred over direct contracting.

## 4.1 Direct Relational Contracts

To analyze direct relational contracts, we take the perspective of buyer *i*. The producer's pre-matching continuation value is assumed to be  $\overline{U} > 0$ . Section 5 endogenizes  $\overline{U}$ .

We say that strategies under a relational contract are *contract-specific* if they do not depend on the player's identity, calendar time, or any history outside the current contract.

<sup>&</sup>lt;sup>13</sup>Ex-post service fees are disallowed for the same reason: buyers can immediately rematch with new managers, so contracts with ex-post service fees are weakly less profitable for buyers.

A relational contract is *stationary* if strategies are time-invariant functions of the buyer's demand realizations. A relational contract is *offerer-optimal* if it yields the highest possible surplus for the party offering the contract. We restrict our attention to offerer-optimal, contract-specific, stationary relational contracts in which producer's effort level is one if and only if  $d_{it} = 1.^{14}$  These contracts must also satisfy the following two conditions. First, on the equilibrium path, parties within a match always choose to continue their relationship immediately after production. Second, off the equilibrium path, deviations are punished in the harshest possible way. These assumptions are standard in the literature (MacLeod and Malcomson, 1998; Baker, Gibbons and Murphy, 2002; Board and Meyer-ter-Vehn, 2015).

Let  $C_i^D = (w_{1i}, w_{0i}, \beta_i)$  denote a contract-specific, stationary relational contract offered by buyer *i* directly to a producer. In this contract,  $w_{1i}$  is the payment when  $d_{it} = 1$ ,  $w_{0i}$  is the payment when  $d_{it} = 0$ , and  $\beta_i \in [0, 1]$  is the probability that buyer *i* stays with the producer when the buyer's demand is zero. The time subscript is dropped since we focus on stationary contracts.

Under a direct contract, the buyer motivates the producer to exert effort using credible promises of future surplus from their contractual relationship. If the buyer deviates from the specified payment, the producer exerts no effort and separates from the buyer after production with probability one. If the producer deviates from the specified effort, the buyer separates from the producer after production with probability one. Since the buyer has fluctuating demand, the retention probability  $\beta_i$  determines the expected duration of the relationship and therefore affects the level of payments needed to incentivize the producer.

If  $\beta_i > 0$ , the post-matching continuation payoffs for the producer when  $d_{it} = 1$  and  $d_{it} = 0$  are, respectively, given by

$$U_{1i} = w_{1i} - c + \delta \Big[ (1 - \alpha_{1i}) U_{1i} + \alpha_{1i} (\beta_i U_{0i} + (1 - \beta_i) \overline{U}) \Big], \tag{1}$$

<sup>&</sup>lt;sup>14</sup>Even though there is limited liability in our model, as shown in Appendix Section A, focusing on stationary relational contracts is without loss since effort is perfectly observable and pairwise stability (or "bilateral efficiency" as in Board and Meyer-ter-Vehn (2015)) is imposed on the equilibrium concept. A pairwise stable relational contract is a Pareto-optimal contract for parties in a match when they take their outside options as given. See Li (2022) for a more detailed discussion on how non-stationary relational contracts may be optimal in equilibrium when the pairwise stability restriction is relaxed.

and

$$U_{0i} = w_{0i} + \delta \Big[ \alpha_{0i} U_{1i} + (1 - \alpha_{0i}) (\beta_i U_{0i} + (1 - \beta_i) \overline{U}) \Big].$$
(2)

The relevant incentive constraints for the producer are as follows:

$$U_{1i} \ge w_{1i} + \delta \overline{U}, \tag{P-IC-e}$$

$$U_{1i} \ge \overline{U},$$
 (P-IC1)

$$U_{0i} \ge \overline{U}.$$
 (P-IC0)

Constraint (P-IC-e) requires the producer to choose effort over shirking when the service is needed. Constraints (P-IC1) and (P-IC0) require that the producer remain with the current buyer when the demand is 1 or 0, respectively. If  $\beta_i = 0$ , equation (2) and constraint (P-IC0) do not apply, as the buyer immediately separates from the producer when demand is zero.

For the buyer, the post-matching continuation payoffs when  $d_{it} = 1$  and  $d_{it} = 0$  are

$$\Pi_{1i} = y - w_{1i} + \delta \Big[ (1 - \alpha_{1i}) \Pi_{1i} + \alpha_{1i} (\beta_i \Pi_{0i} + (1 - \beta_i) \overline{\Pi}_{0i}) \Big],$$

and

$$\Pi_{0i} = -w_{0i} + \delta \left[ \alpha_{0i} \Pi_{1i} + (1 - \alpha_{0i}) (\beta_i \Pi_{0i} + (1 - \beta_i) \overline{\Pi}_{0i}) \right].$$

where  $\overline{\Pi}_{1i}$  and  $\overline{\Pi}_{0i}$  are the buyer's pre-matching continuation values when  $d_{it} = 1$  and  $d_{it} = 0$ , respectively. Since there is an excess of producers in the frictionless matching market, the buyer can always successfully find a match, so  $\overline{\Pi}_{1i} = \Pi_{1i}$  and  $\overline{\Pi}_{0i} = \Pi_{0i}$ .

The relevant incentive constraint for the buyer are:

$$\Pi_{1i} \ge \delta(\alpha_{1i}\overline{\Pi}_{0i} + (1 - \alpha_{1i})\overline{\Pi}_{1i}), \qquad (B-IC-w)$$

$$\Pi_{1i} \ge \Pi_{1i}, \tag{B-IC1}$$

$$\Pi_{0i} \ge \overline{\Pi}_{0i}. \tag{B-IC0}$$

Constraint (B-IC-w) ensures that the buyer honors the payment to the producer. Constraints (B-IC1) and (B-IC0) reflect the buyer's desire to retain the producer when there is a demand or not, respectively. As before, if  $\beta_i = 0$ , the term  $\Pi_{0i}$  and constraint (B-IC0) do not apply, as the buyer would immediately separate from the producer.

The optimal direct contract is obtained by choosing  $w_{1i}$ ,  $w_{0i}$ , and  $\beta_i$  to maximize  $\Pi_{1i}$ , subject to (B-IC-w), (B-IC1), (P-IC-e), (P-IC1), as well as (B-IC0) and (P-IC0) if  $\beta_i > 0$ .

Lemma 1. Suppose an optimal direct relational contract exists. Under this contract, if

$$\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_{0i}}{1-\alpha_{1i}},\tag{3}$$

then the buyer pays

$$w_{DS}(\alpha_i) = \left(\frac{1}{\delta} \frac{1}{1 - \alpha_{1i}}\right) c + (1 - \delta)\overline{U}.$$
(4)

when demand is one and separates from the producer when demand becomes zero. Otherwise, the buyer pays

$$w_{DI}(\alpha_i) = \left(\frac{1}{\delta} \frac{1 + \frac{\delta}{1 - \delta} \alpha_{0i}}{(1 - \alpha_{1i}) + \frac{\delta}{1 - \delta} \alpha_{0i}}\right) c + \left(\frac{1 + \frac{\delta}{1 - \delta} \alpha_{0i}}{(1 - \alpha_{1i}) + \frac{\delta}{1 - \delta} \alpha_{0i}} - \delta\right) \overline{U}$$
(5)

when demand is one, remains matched with the idle producer when demands switches to zero, and pays zero until demand switches back to one.

*Proof.* All omitted proofs are in the appendix.

Lemma 1 shows that the optimal direct contract features a buyer who either always separates from a producer or always retains him when their demand switches to zero. According to Equation (3), direct contracts with separation dominates direct contracts with idleness when (a) the producer's continuation value when unmatched  $\overline{U}$  is high, (b) the buyer has a smaller  $\alpha_{0i}$ , so a longer period of idleness is expected, and (c) the buyer has a larger  $\alpha_{1i}$ , so a shorter period with service demand is expected.

The payment to producers in a direct contract,  $w_D(\alpha_i) \equiv \min\{w_{DS}(\alpha_i), w_{DI}(\alpha_i)\}$ , is

increasing in  $\alpha_{1i}$ . This is because when business needs are shorter-lived, the producer faces a higher chance of either separating or becoming idle, so the future surplus in the relationship is smaller. A higher incentive rent is therefore needed to incentivize producer effort.

# 4.2 Managed Relational Contracts

Under managed contracts, buyers delegate to managers the responsibility of motivating and monitoring producers. The manager fulfills the buyer's demand by entering relational contracts with a large number of producers and assigning producers to buyers according to demand realization.

We first analyze the contract that managers offer to producers. To meet buyer demand, we assume that each manager offers offerer-optimal, contract-specific, and stationary contracts to producers. As shown in Section 4.1, the terms in an optimal direct contract with producer hinge on the buyer's demand-switching probabilities. Unlike buyers, however, managers face constant demand for services and therefore have constant demand for effort from their matched producers. The reason is that each manager randomly matches with a continuum of buyers drawn from the same distribution, so by the law of large numbers, the managers face total demand from buyers that is constant over time. Anticipating this stable demand for services, the measure of producers that each manager contracts with is equal to the expected measure of demand realizations, and each matched producer is asked to exert effort in every period. Therefore, by the logic of Lemma 1, the compensating payment for the producer is  $w_{1i} = w_M$  in every period, where

$$w_M = \frac{c}{\delta} + (1 - \delta)\overline{U}.$$
(6)

We next consider how producers are assigned to buyers in each period under the managed contract. Note that buyers are indifferent between any assignment of producers where the assigned producer exerts effort, since producers are identical in our model. Producers are also indifferent between any assignment of buyers where the managers offer the same level of payment. Since managers require producers to exert effort and provide the same compensating payment in every period, buyers and producers have the same payoffs in any assignment where producers are matched with buyers with positive demand in every period. There are an infinite number of such assignments, and any such assignment is optimal.

We now characterize the contract that buyers offer to managers. Let  $C_i^M = (p_{1i}, p_{0i}, \beta_i)$  denote a buyer-optimal, contract-specific, and stationary managed relational contract offered by a buyer to a manager. Here  $p_{1i}$  and  $p_{0i}$  are the service fees when the buyer needs and does not need the service, respectively. If the buyer deviates from the specified service fee, the manager does not assign a producer to the buyer and separates from the buyer. If instead the producer assigned by the manager deviates from the specified effort, the buyer separates from the manager after production.

Under  $C_i^M$ , the post-matching continuation payoffs for the manager in a buyer-manager match, when the service is and is not needed, respectively, are

$$V_{1i} = p_{1i} - w_M + \delta \left[ (1 - \alpha_{1i}) V_{1i} + \alpha_{1i} (\beta_i V_{0i} + (1 - \beta_i) \overline{V}) \right], \tag{7}$$

and

$$V_{0i} = p_{0i} + \delta \Big[ \alpha_{0i} V_{1i} + (1 - \alpha_{0i}) (\beta_i V_{0i} + (1 - \beta_i) \overline{V}) \Big],$$
(8)

where  $\overline{V}$  is a manager's continuation value after separating from a buyer. The relevant incentive constraints for the manager, similar with those for a producer, are

$$V_{1i} \ge p_{1i} + \delta \overline{V}, \tag{M-IC-w}$$

$$V_{1i} \ge \overline{V},$$
 (M-IC1)

$$V_{0i} \ge \overline{V}.$$
 (M-IC0)

Note that if a manager separates from a buyer, it cannot match with a new buyer, because all other potential buyers are matched with some manager and will not become unmatched on the equilibrium path. The manager also cannot re-match with the initial buyer, since manager-buyer matching is assumed to not be anonymous. Therefore,  $\overline{V} = 0$  on the equilibrium path.

For the buyer, the continuation payoffs and incentive constraints are the same as in the direct contract, except that the payments  $w_{0i}$  and  $w_{1i}$  to the producer are replaced with service fee payments  $p_{0i}$  and  $p_{1i}$  to the manager. For concision, we omit these conditions, which simply repeat (B-IC-w), (B-IC-1), and (B-IC0). The optimal managed contract maximizes  $\Pi_{1i}$  subject to these incentive compatibility constraints.

**Lemma 2.** Suppose an optimal managed contract exists. Under this contract, the buyer always retains the manager, pays zero service fees to the manager when there is no demand, and when there is demand, she pays the manager a service fee equal to

$$p(\alpha_i) = \left(\frac{1}{\delta} \frac{1 + \frac{\delta}{1 - \delta} \alpha_{0i}}{(1 - \alpha_{1i}) + \frac{\delta}{1 - \delta} \alpha_{0i}}\right) w_M.$$
(9)

Lemma 2 shows that the cost of managed contract is a form of double marginalization. Note that manager pay  $p(\alpha_i)$  is the product of  $\lambda(\alpha_i) = \frac{1}{\delta} \frac{1 + \frac{\delta}{1-\delta} \alpha_{0i}}{(1-\alpha_{1i}) + \frac{\delta}{1-\delta} \alpha_{0i}}$  and producer pay  $w_M$ . Here  $\lambda(\alpha_i)$  can be thought of as a manager markup. Furthermore, as shown in Equation (6),  $w_M$  is elevated above the producer's cost of effort. In other words, both the manager and producer are paid rents so that both are incentivized to honor their contractual obligations.

The benefit of managed contract is that the payment to the producer is lower than the payment under direct contracts, since the manager can smooth demand across its buyers. To see this, note that  $w_D(\alpha_i) \ge w_M$  for all  $\alpha_i$ , where equality holds if and only if  $\alpha_{1i} = 0$ .

## 4.3 **Optimal Contractual Choice**

Having characterized direct and managed contracts, we now characterize when buyers choose managed contracts. We focus on buyers who have the same  $\alpha_{0i}$ , which is the rate at which a buyer's demand changes from zero to one. We examine how the optimal contractual choice depends on  $\alpha_{1i}$ , the rate at which the buyer's demand changes from one to zero, and  $\overline{U}$ , the producer's outside option, which in equilibrium reflects whether the producer market is tight



Figure 4: Service cost under direct and managed contracts

Note:  $\alpha_1$  is the probability that the buyer's demand switches from 1 to 0.

and whether it is easy for producers to rematch.

To compare the two, it suffices to compare the payment  $w_D(\alpha_i)$  under direct contracting and the service fee  $p(\alpha_i)$  under managed contracting. This is because the buyer pays nothing when their demand is zero and can always match with a producer when her demand is one.

Figure 4 provides a graphical illustration. For a buyer with stable demand, i.e., for whom  $\alpha_{1i} = 0$ , managed contracting is strictly more expensive than direct contracting because of double marginalization. To obtain high-quality services, the buyer needs to pay additional rent to the manager. However, there is no benefit to managed contracts, since demand is stable, so the producer is paid the same incentive rent under direct contracting.

The advantage of managed contracting over direct contracting becomes larger when business needs are short-term. As  $\alpha_{1i}$  becomes larger, producers must be paid elevated payments in order for them to exert effort. Since managers can reassign producers across buyers, producers never separate or become idle, so the cost of incentivizing producers is lowered. To see this mathematically, note that  $w_{DS}(\alpha_i)$  approaches infinity as  $\alpha_{1i}$  approaches one. Furthermore,  $p(\alpha_i)$  increases in  $\alpha_{1i}$  less steeply than  $w_{DI}(\alpha_i)$  if c is relatively small and  $\overline{U}$  is relatively large. Therefore, when rematching is easy, the manager operates a more Figure 5: Optimal contractual choice given model parameters



cost-efficient internal producer market by having long-standing relational contracts on both sides of the market.

Figure 5 graphically shows the optimal contracts as a function of  $\alpha_{1i}$  and  $\overline{U}$ . In this figure, provided  $\overline{U} > \overline{U}^*$ , then there exists a cutoff value such that managed contracts dominate if and only if  $\alpha_{1i}$  is sufficiently large. If  $\overline{U} < \overline{U}^*$ , then the producer's pre-matching continuation value is low, so it is optimal to retain them and keep them idle until demand returns.

The following proposition formalizes this result.

**Proposition 1.** Take any set of buyers *i* with the same  $\alpha_{0i}$  for whom an optimal contract exists. There exists  $\overline{U}^* > 0$  such that:

- 1. If  $\overline{U} < \overline{U}^*$ , a direct contract is optimal for all *i* in this set;
- 2. If  $\overline{U} > \overline{U}^*$ , there exists  $\alpha_1^*(\alpha_{0i}, \overline{U}) \in (0, 1)$  such that a managed contract is optimal if and only if  $\alpha_{1i} \ge \alpha_1^*(\alpha_{0i}, \overline{U})$ .

Proposition 1 can be viewed a microfounded answer to a variant of the question first posed by Coase (1937): Why and when should we expect centralized resource allocators to emerge in decentralized *relational contracting* markets, even when buyers and producers can frictionlessly meet, possess no hidden information about capabilities, and can freely enter contracts? The result shows that managerial coordination dominates direct pairings when

business needs are short-term and the continuation value of an unmatched producer is high. An important implication is that centralized allocators may emerge even in the absence of search, bargaining, and contract-writing costs.

# 5 Steady-State Equilibrium

In this section, we show that there exists a unique steady-state equilibrium. We then explore the model's empirical implications. First, we compare the outcomes of managed producers and directly contracted producers in equilibrium. Second, we compare the outcomes and welfare of buyers and producers in economies with and without managers.

## 5.1 Deriving the Equilibrium

We say that the economy is in a steady state when (1) the number of producers in the economy and the distribution of contracts in the matching market are unchanging across periods and (2) each buyer's demand evolves according to its stationary distribution. By the properties of Markov processes, the steady-state probability that a buyer *i* has positive demand in any period is equal to  $\pi_i = \alpha_{0i}/(\alpha_{0i} + \alpha_{1i})$ .

At the start of each period, some buyers and managers are unmatched and directly offer direct contracts to unmatched producers. Let  $\mathcal{I}_{DS}$  denote the set of buyers who enter direct contracts that end when their demand switches to zero. For a buyer  $i \in \mathcal{I}_{DS}$ , the steady-state probability that they offer new contracts is

$$v_i = (1 - \pi_i)\alpha_{0i}.$$
 (10)

Let  $I_{DI}$  be the set of buyers who directly contract with producers in contracts that continue when demand switches. Let  $I_M$  be the set of buyers who contract with managers. For both types of contracts, producers never separate. Therefore, for any buyer  $i \in I_{DI} \cup I_M$ , the steady-state probability that they offer new contracts is  $v_i = 0$ . The total measure of new contracts offered by buyers and managers in the producer market is given by

$$v = \int_{\mathcal{I}_{DS}} v_i dF. \tag{11}$$

The steady-state measure of producers in direct contracts,  $n_B$ , is given by

$$n_B = \int_{\mathcal{I}_{DS}} \pi_i dF + \int_{\mathcal{I}_{DI}} dF.$$
(12)

The steady-state measure of producers in managed contracts,  $n_M$ , is given by<sup>15</sup>

$$n_M = \int_{I_M} \pi_i dF. \tag{13}$$

The measure of unmatched producers after matching is

$$n_N = n - n_B - n_M. \tag{14}$$

If y and C are sufficiently large and the buyer type distribution F has full support on  $[0, 1] \times [0, 1]$ , the value of entering either direct or managed contracts is higher than the value of being unmatched. This implies that there is an excess of producers who enter the producer market, so  $n_N > 0$ .

Since matching is frictionless, all contracts offered in the producer market are immediately filled. The total measure of unmatched producers before matching, u, is given by

$$u = v + n_N. \tag{15}$$

Matching is random and contract offers are never made to matched producers, so the Bellman equation for an unmatched producer is given by

$$\overline{U} = \int_{\mathcal{I}_{DS}} \frac{v_i}{u} U_{DS}(\alpha_i) dF + \left(1 - \frac{v}{u}\right) \overline{U},\tag{16}$$

<sup>&</sup>lt;sup>15</sup>Here  $\pi_i$  enters the integral since the manager aggregates the fluctuating demand of buyers.

where  $U_{DS}(\alpha_i) = \frac{1}{1-\delta(1-\alpha_{1i})} (w_{DS}(\alpha_i) - c + \delta \alpha_{1i} \overline{U})$ . Note that the continuation value of being unmatched ( $\overline{U}$ ) consists of two components. The first term reflects the rent from matching with an unmatched buyer. The second term reflects the value of remaining unmatched.

To complete the model, we solve for the steady-state number of producers that enter the economy. By assumption, producers enter the economy at the beginning of each period by paying an entry cost C. Entry drives down the likelihood that producers are matched, so they enter only until the continuation value of being unmatched in the labor market equals their entry cost. This yields the following condition:

$$\overline{U} = C. \tag{17}$$

We can now define an equilibrium in our economy.

**Definition 1.** A steady-state equilibrium is a distribution of relational contracts offered by each buyer and manager such that:

- 1. All relational contracts are offerer-optimal, contract-specific, and stationary;
- 2. Each player's pre-matching continuation value is determined by steady-state transition probabilities and frictionless and random matching via Equation (16);
- 3. The measure of producers in the economy is derived from the producer entry condition, given by Equation (17).

**Proposition 2.** There exists a unique steady-state equilibrium. If y and C are sufficiently high and F has full support on  $[0, 1] \times [0, 1]$ , then a non-zero measure of buyers enter direct contracts, while another non-zero measure of buyers enter managed contracts.

*Proof.* By Equation (17),  $\overline{U}$  equals the entry cost *C*. Given  $\overline{U}$ , we can compare the values of  $w_{DS}(\alpha_i)$ ,  $w_{DI}(\alpha_i)$ ,  $p(\alpha_i)$ , and *y* for each *i* using Equations (4), (5), and (9) to determine  $I_{DS}$ ,  $I_{DI}$ , and  $I_M$ . Having derived these, we can obtain unique values for  $v_i$ , v,  $n_B$ , and  $n_M$  from Equations (10), (11), (12), and (13). We plug these into Equation (16) to solve for a

unique value for u. Plugging u into Equations (14) and (15) then yields a unique value for n. The desired statement then follows from Proposition 1.

# **5.2 Empirical Implications**

Having shown that there exists a unique steady-state equilibrium in this exchange economy, we can now explore the model's predictions regarding equilibrium behavior.

#### **Effects of Managers on Producers**

How does managerial coordination affect the pay, separation rates, and idleness of producers? We focus on the interesting case where all three contract types arise in equilibrium.<sup>16</sup> We define a producer's *separation rate* as the probability that a matched producer becomes unmatched at the start of the next period. We define *idleness* as the steady-state probability that a producer is matched but does not exert effort. We derive four findings.

**Corollary 1.** Directly contracted producers have (1) higher average pay, (2) more dispersed pay, (3) higher separation rates, and (4) higher idleness than managed producers.

The first result follows from the fact that  $w_D(\alpha_i) > w_M$  for all buyers *i* with fluctuating demand. The second follows from the fact that  $w_D(\alpha_i)$  takes on different values depending on  $\alpha_i$ , which is heterogeneous across buyers, while  $w_M$  is constant. The final two follow from the fact that directly contracted producers may separate from buyers or become idle when demand changes, while managed producers are always reallocated among the manager's clients, so they never separate nor become idle.

The predictions broadly align with recent evidence. Workers managed by professional service firms earn lower and more compressed wages (Dube and Kaplan 2010; Goldschmidt and Schmieder 2017; Drenik et al. 2023). Some evidence also suggests that they have lower hazard into unemployment than comparable direct employees (Guo, Li and Wong 2024).

<sup>&</sup>lt;sup>16</sup>Specifically, we assume that y, C and F are such that  $|I_M|$ ,  $|I_{DI}|$ ,  $|I_{DS}| > 0$ . If F has full support on  $[0, 1] \times [0, 1]$ , then by Proposition 1 there exists y and C such that this holds.

#### Trade and Welfare with and without Managers

How does the presence of managers alter market equilibrium and the welfare of buyers and producers? Felix and Wong (2024) show that Brazil's 1993 legalization of professionally managed security firms persistently increased security guard employment, but displaced incumbent security guards from high-wage direct employers.

To explore the same question theoretically, we consider two economies: one with managers and one without. The introduction of managers causes three types of buyers to switch to managed contracts: buyers who do not initially consume services, buyers who initially choose direct contracts with idleness, and buyers who initially choose direct contracts with idleness, and buyers who initially choose direct contracts with separation. We denote the three subsets of switching buyers as  $S_0$ ,  $S_I$ , and  $S_S$ . We assume that there is a positive measure of switchers who initially do not consume, and that there is a positive measure of switchers who directly contract initially.<sup>17</sup>

In both economies, producers are assumed to freely enter at cost C. As such, the introduction of managers does not alter the payments received by producers who remain directly matched to same buyers. It only reduces the payments received by producers who are become matched to managers instead of buyers. It follows that:

**Corollary 2.** When managers are introduced into an economy, the payoffs of all buyers weakly increase and the measure of buyers who receive services increases. The mean pay per unit effort received by producers falls. The total measure of matched producers increases if and only if  $\int_{S_0} \pi_i dF > \int_{S_1} (1 - \pi_i) dF$ .

These predictions are intuitive. The introduction of managers benefits buyers with fluctuating demand, but it hurts incumbent producers who were initially earning pay premiums. As such, it unambiguously increases the measure of buyers who receive services. However, it is ambiguous whether the measure of matched producers  $(n_M + n_B)$  increases or falls. On one hand, managers enable more buyers to afford services, which increases demand

<sup>&</sup>lt;sup>17</sup>In other words, y, C and F are such that  $|S_0| > 0$  and  $|S_I \cup S_S| > 0$ . Assuming indifferent buyers do not switch, this requires that there exists  $\alpha_i, \alpha'_i \in \text{supp } F$  such that  $y > w_D(\alpha_i) > p(\alpha_i)$  and  $w_D(\alpha'_i) > y > p(\alpha'_i)$ . By Proposition 1, if F has full support on  $[0, 1] \times [0, 1]$ , then there exist y and C such that this holds.

for producers. On the other, managers reduce idleness, so fewer producers are needed. The relative magnitude of these two effects depends on the distribution of buyer types.

Online Appendix B analyzes the case where the producer population is fixed instead. In this alternative model, the introduction of managers similarly alters the demand for producers, but will instead alter the payoffs of all producers in the economy.

# 6 Extensions

This section extends the model to study the relationship between managerial coordination, producer specialization, and manager reputation concerns. We first incorporate an endogenous choice by producers to specialize in different capabilities. Then, we add the possibility that poor performance by a manager is made known to a broader set of buyers.

## 6.1 Specialization

We consider a multi-task economy with a measure of buyers who have unit demand in every period, for either one of two tasks  $d_{it} \in \{A, B\}$ . Each buyer's demand switches from one task to another task with some symmetric probability  $\alpha_i$  at the start of each period.<sup>18</sup> There is also an excess measure of producers indexed by j who choose whether to become either specialists in one of two activities needed by buyers or a generalist with middling skill in both services. Let  $\phi_j \in \{A, B, G\}$  denote the chosen type of the producer, where A and Brefer to specialists and G refers to the generalist. The output depends on the buyer's demand  $d_{it}$ , the producer's type  $\phi_j$ , and the producer's chosen effort  $e_{jt}$ , and is given by

$$y_{ijt} = \left[ y \cdot \mathbf{1} \{ \phi_j = G \} + (y + \Delta_i) \cdot \mathbf{1} \{ \phi_j = d_{it} \} \right] e_{jt},$$

where  $\Delta_i > 0$  denotes the buyer-specific *gains from specialization*. If the specialist exerts effort, output is high when demand and the producer's type are well-matched, but low when

<sup>&</sup>lt;sup>18</sup>This is essentially simplifying the model in Section 3 by assuming  $\alpha_{Ai} = \alpha_{Bi} = \alpha_i$ .

Figure 6: Optimal contractual choice in the presence of gains from specialization Gains from specialization  $\Delta_i$ 



they are not. Output is always middling for generalists who exert effort.

As before, neither pay w nor effort e are contractible and must be incentivized through relational contracts. We assume that output y is sufficiently large so that buyers are always able to receive positive profit by directly contracting with a generalist, so there are never buyers who do not receive services. We also assume that each producer's entry cost C is sufficiently large so that specialist producers never remain in a contract but become idle when the demand of their buyer changes. These assumptions allow us to focus on each buyer's choice between direct contracting with a generalist, direct contracting with a specialist who is never idle, and managed contracting. There are K managers for each task. Each manager can only monitor producers specializing in that task and are randomly matched with buyers who offer managed contracts.<sup>19</sup>

#### Drivers of Managerial Coordination in a Multi-task Economy

Which buyers choose managed contracts in a multi-task economy? Abraham and Taylor (1996) find that establishments with cyclical demand and specialized needs are more likely to outsource accounting services to professionally managed firms. Similar patterns are found in our model, as visualized in Figure 6 and stated formally in the following proposition.

<sup>&</sup>lt;sup>19</sup>This setup implicitly assumes that each manager specializes in monitoring one type of task.

**Proposition 3.** A unique steady-state equilibrium exists in a multi-task economy. Buyer *i* chooses managed contracts if and only if their demand volatility  $\alpha_i$  and gains from specialization  $\Delta_i$  are both sufficiently large.

#### **Division of Labor with or without Managers**

How does the presence of managers affect the division of labor? To answer this question, we compare multi-task economies with and without managers. We assume that if managers are present, a non-zero measure of buyers choose managed contracts and the conditional distribution of  $\alpha_i$  given  $\Delta_i$  has positive support on [0, 1].

**Corollary 3.** When managers are introduced into a multi-task economy, the measure of specialist producers increases and the measure of unmatched producers falls.

The intuition is as follows. The dashed curve in Figure 6 shows the boundary between direct contracting with a specialist and with a generalist when managers are absent. When managers are present, part of the demand for directly contracted generalists is replaced with demand for managed specialists. Overall demand for specialists rises. In response, more producers choose to become specialists. Correspondingly, there are fewer direct contracts with elevated pay, so fewer producers enter, and the measure of unmatched producers falls.

## 6.2 Manager Reputational Concerns

Thus far, our model follows Shapiro and Stiglitz (1984) in assuming that producers are motivated to perform through the threat of contract termination. Another source of motivation, as modeled by Klein and Leffler (1981), is that producers may lose reputational capital when they renege on promises to deliver high-quality services. In this subsection, we study how manager reputation concerns shape the choice between managed and direct contracting.

We consider a multi-task economy with reputable managers, where low effort by a producer is communicated with some probability to other buyers who can withhold future business from the producer's matched manager. Specifically, we assume that with probability  $\gamma \in [0, 1]$ , the effort choice by a producer contracted by the manager is observed by another buyer, who is drawn among all buyers with uniform probability.<sup>20</sup>

#### Communication Technology, Manager Coordination, and the Division of Labor

How does the arrival of communication technologies like the Internet and social media affect the extent of managerial coordination and the division of labor? The parameter  $\gamma$  measures the *ease of word-of-mouth communication* about manager performance as enabled by communication technologies. A manager's mean continuation value from being matched with a buyer is therefore  $\tilde{V} = \frac{1}{|I|} \int_{I_O} \pi_i V_{1i} + (1 - \pi_i) V_{0i} dF > 0$ . When  $\gamma > 0$ , the manager's binding IC constraint becomes

$$V_1 \ge p_1 + \delta(-\gamma \tilde{V}).$$
 (M-IC-w')

As  $\gamma$  increases, the manager faces a harsher threat of multilateral punishment. Therefore, a reduced mark-up is needed to incentivize the manager to perform. Consequently, the unit cost of managed service decreases as the ease of communication increases. This in turn increases managerial coordination, and thereby enables specialization and reduces the measure of unmatched producers.

**Proposition 4.** A unique steady-state equilibrium exists in a multi-task economy with reputable managers. As the ease of word-of-mouth communication increases, the measure of managed producers increases, the measure of specialist producers increases, and the measure of unmatched producers falls.

These predictions help make sense of recent macroeconomic trends. Bergeaud et al. (Forthcoming) provide causal evidence that the rise of broadband internet increased the employment share of professional service firms and the average occupational concentration of firms. Correlational evidence shows that as the employment share of professional service

<sup>&</sup>lt;sup>20</sup>Here the manager will not wish to renege on more than one of its clients if she does not wish to do so for a single client, since a buyer may learn of bad service provided to multiple clients from word-of-mouth communication but can only punish maximally once.

firms has increased, workers and firms have become increasingly specialized, and that unemployment has fallen (Katz and Krueger 1999; Song et al. 2019; Handwerker 2023).

# 7 Conclusion

Economists have long puzzled over why managers often replace markets in allocating resources. This paper sheds new light on the puzzle by developing a general equilibrium model of an exchange economy with imperfect contractual enforcement, in which resourceallocating managers endogenously emerge as central nodes in relational contracting markets. In the model, managers emerge to alleviate a tension between incentive provision and flexible allocations in fluctuating environments. The use of managers involves a trade-off between demand aggregation and double marginalization. Consequently, managers only partially replace anonymous matching markets in determining assignments between buyers and producers. In equilibrium, buyers sort into managerial coordination and decentralized exchange depending on their demand volatility, their gains from specialization, and market-level communication frictions. The presence of managers alters aggregate output, the division of labor, matching market flows, and the distribution of economic rents.

A key finding is that centralized monitor-allocators can emerge in exchange economies in the absence of many factors previously hypothesized to be important—including contractwriting costs, bargaining frictions, search frictions, and adverse selection. Another is that managerial coordination may increase as communication frictions fall. The predictions of the model are not only applicable to a wide variety of organizational forms, they are also realistic when compared to recent empirical findings on professional service firms. We conclude that formal analysis of centralized relational contracting structures, as attempted in this paper, is a fruitful avenue for shedding new light on the microeconomic logic and equilibrium behavior of real-world organizations.

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# A Appendix

We omit the subscript *i* in the proofs. We first establish the following two lemmas.

**Lemma A.1.** Suppose an optimal bilateral relational contract exists. Under this contract, if  $\beta > 0$ , then the following three conditions hold: 1)  $w_0 = 0$ , 2) both (B-IC1) and (B-IC0) bind, and 3) (P-IC1) is slack.

*Proof.* Prove by contradiction. Suppose that  $w_0 > 0$ . Since there is an excess of producers in the matching market,  $\overline{\Pi}_1 = \Pi_1$ . So  $\Pi_0 = -w_0 + \delta \left[ \alpha_0 \Pi_1 + (1 - \alpha_0) (\beta \Pi_0 + (1 - \beta) \overline{\Pi}_0) \right] = -w_0 + \delta \left[ \alpha_0 \overline{\Pi}_1 + (1 - \alpha_0) (\beta \Pi_0 + (1 - \beta) \overline{\Pi}_0) \right]$ . Also note that  $\overline{\Pi}_0 = \delta \left[ \alpha_0 \overline{\Pi}_1 + (1 - \alpha_0) \overline{\Pi}_0 \right]$ . Therefore,  $\Pi_0 - \overline{\Pi}_0 = -w_0 + \delta(1 - \alpha_0)\beta(\Pi_0 - \overline{\Pi}_0) < \delta(1 - \alpha_0)\beta(\Pi_0 - \overline{\Pi}_0)$ , where the inequality comes from  $w_0 > 0$ . Since  $\delta \in (0, 1)$  and  $\beta \in (0, 1]$ , we discuss whether  $\alpha_0 = 1$ . If  $\alpha_0 < 1$ , the inequality cannot be satisfied, contradicting  $w_0 > 0$ . If  $\alpha_0 = 1$ ,  $\overline{\Pi}_0 = \delta \overline{\Pi}_1$  and then  $\Pi_0 = -w_0 + \delta \overline{\Pi}_1 < \overline{\Pi}_0$ , which contradicts (B-IC0) and also indicates that  $w_0 = 0$ . Therefore,  $w_0 = 0$  and  $\Pi_0 = \overline{\Pi}_0$ . Thus both (B-IC1) and (B-IC0) bind.

We now show that (P-IC1) is slack. Suppose it binds, so  $U_1 = \overline{U}$ . Then given  $w_0 = 0$  and  $\delta < 1$ , plug in  $U_1 = \overline{U}$  and get  $U_0 = \delta \left[ \alpha_0 \overline{U} + (1 - \alpha_0)(\beta U_0 + (1 - \beta)\overline{U}) \right] < (1 - \alpha_0)\beta U_0 + (1 - (1 - \alpha_0)\beta)\overline{U}$ . This inequality implies that  $U_0 < \overline{U}$ , which contradicts (P-IC0). So (P-IC1) is slack.

**Lemma A.2.** For any buyer who directly contracts with producers, maximizing  $\Pi_1$  is equivalent to minimizing  $w_1$ .

*Proof.* Based on Lemma A.1, the buyer's continuation payoffs can be written as  $\Pi_1 = \frac{1-\delta(1-\alpha_0)}{1-\delta\alpha_0\alpha_1-\delta(2-\alpha_0-\alpha_1)+\delta^2(1-\alpha_0)(1-\alpha_1)}(y-w_1)$ , and  $\Pi_0 = \frac{\delta\alpha_0}{1-\delta(1-\alpha_0)}\Pi_1$ . Since  $\frac{\partial\Pi_1}{\partial w_1} < 0$ , a buyer's problem is equivalent to minimize  $w_1$  subject to producer's incentive constraints.

# Proof of Lemma 1

For simplicity, we write  $w_1$  as w in the rest of the proof. We complete the proof by analyzing and comparing the terms in the optimal bilateral relational contracts when choosing  $\beta = 0$  or  $\beta > 0$ .

**Choice 1:**  $\beta = 0$ . If the optimal choice of  $\beta$  is 0,  $U_1 = w - c + \delta \left[ (1 - \alpha_1) U_1 + \alpha_1 \overline{U} \right]$ . The buyer optimally chooses *w* subject to a binding (P-IC-e), which gives  $w_{DS} = \frac{1}{\delta(1 - \alpha_1)}c + (1 - \delta)\overline{U}$ .

**Choice 2:**  $\beta \in (0, 1]$ . If the optimal choice of  $\beta$  is greater than 0, a buyer's optimization problem becomes  $\min_{w,\beta} w$  subject to (P-IC-e), (P-IC0), (1), (2),  $0 < \beta \le 1$ , and  $w \ge 0$ .

The Lagrangian is given by

$$\begin{split} L &= w + \lambda_1 (U_1 - (w - c + \delta((1 - \alpha_1) \cdot U_1 + \alpha_1(\beta U_0 + (1 - \beta)\overline{U})))) \\ &+ \lambda_2 (U_0 - \delta(\alpha_0 \cdot U_1 + (1 - \alpha_0) \cdot (\beta U_0 + (1 - \beta)\overline{U}))) \\ &+ \mu_1 (w + \delta \overline{U} - U_1) + \mu_2 (\overline{U} - U_0) + \mu_3 (\beta - 1) + \mu_4 (-w). \end{split}$$

The Kuhn-Tucker conditions are given by  $\frac{\partial L}{\partial w} = 1 + \mu_1 (1 - \frac{\partial U_1}{\partial w}) - \mu_2 \frac{\partial U_0}{\partial w} - \mu_4 \le 0; \frac{\partial L}{\partial w} w = 0;$  $\frac{\partial L}{\partial \beta} = -\mu_1 \frac{\partial U_1}{\partial \beta} - \mu_2 \frac{\partial U_0}{\partial \beta} - \mu_3 = 0; \lambda_1, \lambda_2 > 0 \ \mu_1, \mu_2, \mu_3, \mu_4 \ge 0; \ \mu_1 (w + \delta \overline{U} - U_1) = 0;$  $\mu_2 (\overline{U} - U_0) = 0; \ \mu_3 (1 - \beta) = 0; \text{ and } \mu_4 w = 0.$ 

By taking derivatives on both sides of equations (1) and (2) with respect to w, we get

$$\frac{\partial U_1}{\partial w} = 1 + \delta \Big[ (1 - \alpha_1) \frac{\partial U_1}{\partial w} + \alpha_1 \beta \frac{\partial U_0}{\partial w} \Big], \tag{A1}$$

$$\frac{\partial U_0}{\partial w} = \delta \left[ \alpha_0 \frac{\partial U_1}{\partial w} + (1 - \alpha_0) \beta \frac{\partial U_0}{\partial w} \right]. \tag{A2}$$

By taking derivatives on both sides of equations (1) and (2) with respect to  $\beta$ , we get

$$\frac{\partial U_1}{\partial \beta} = \delta \Big[ (1 - \alpha_1) \frac{\partial U_1}{\partial \beta} + \alpha_1 (U_0 - \overline{U} + \beta \frac{\partial U_0}{\partial \beta}) \Big], \tag{A3}$$

$$\frac{\partial U_0}{\partial \beta} = \delta \Big[ \alpha_0 \frac{\partial U_1}{\partial \beta} + (1 - \alpha_0) (U_0 - \overline{U} + \beta \frac{\partial U_0}{\partial \beta}) \Big].$$
(A4)

We proceed by the following steps.

**Step 1: Show that**  $\mu_4 = 0$  **and** w > 0. Suppose that w = 0. Then

$$U_{1} = -c + \delta \left[ (1 - \alpha_{1})U_{1} + \alpha_{1}(\beta U_{0} + (1 - \beta)\overline{U}) \right] < (1 - \alpha_{1})U_{1} + \alpha_{1}(\beta U_{0} + (1 - \beta)\overline{U}),$$

where the inequality comes from that c > 0 and  $\delta < 1$ . The inequality above implies that  $U_1 < \beta U_0 + (1 - \beta)\overline{U} < U_0$ , where the second inequality comes from (P-IC0). Meanwhile,

$$U_0 = \delta \left[ \alpha_0 U_1 + (1 - \alpha_0) (\beta U_0 + (1 - \beta) \overline{U}) \right] < \alpha_0 U_1 + (1 - \alpha_0) (\beta U_0 + (1 - \beta) \overline{U}) < U_0,$$

where the first inequality comes from  $\delta < 1$  and the second inequality comes from (P-IC0) and  $U_1 < U_0$ . Since  $U_0 < U_0$  can never be true, we know that w > 0 and thus  $\mu_4 = 0$ .

The implication for w > 0 is that  $\frac{\partial L}{\partial w} = 1 + \mu_1 (1 - \frac{\partial U_1}{\partial w}) - \mu_2 \frac{\partial U_0}{\partial w} = 0$ .

**Step 2: Discuss the values of**  $\mu_1$  **and**  $\mu_2$ . **Case 1**)  $\mu_1 = \mu_2 = 0$ . In this case,  $\frac{\partial L}{\partial w} = 0$  is violated, indicating that this case is not possible. In other words, at least one of two incentive compatibility constraints bind.

**Case 2**)  $\mu_1 = 0$  and  $\mu_2 > 0$ . In this case,  $U_1 > w + \delta \overline{U}$  and  $U_0 = \overline{U}$ . Solve  $U_1$  and w based on equation (1), and we get  $U_1 = \frac{1 - \delta(1 - \alpha_0)}{\delta \alpha_0} \overline{U}$ , and  $w = c + \frac{(1 - \delta)^2 - \delta(1 - \delta)(\alpha_0 + \alpha_1)}{\delta \alpha_0} \overline{U}$ . So  $\frac{\partial U_1}{\partial \beta} = \frac{\partial U_0}{\partial \beta} = 0$ . Meanwhile,  $\frac{\partial L}{\partial w} = 0$  and  $\mu_1 = 0$  imply that  $1 = \mu_2 \frac{\partial U_0}{\partial w}$ .  $\frac{\partial L}{\partial \beta} = 0$  implies  $\mu_3 = 0$ , namely  $\beta < 1$ .

There are two conditions that need to be satisfied for this case to be feasible and optimal. First, for feasibility, the solved w and  $U_1$  need to satisfy  $U_1 > w + \delta \overline{U}$ . After plugging in  $U_1$  and w as functions of  $\overline{U}$ , this requires that  $\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_0}{1-\alpha_1}$ . Second, since we are considering the case where choosing  $\beta > 0$  is weakly better than choosing  $\beta = 0$ , the solved w needs to be lower than  $w_{DS}$ , which gives  $\frac{(1-\delta)\overline{U}}{c} \le \frac{\alpha_0}{1-\alpha_1}$ . These two conditions contract each other, indicating that this case is not possible.

**Case 3**)  $\mu_1 > 0$  and  $\mu_2 > 0$ . In this case, both incentive compatibility constraints bind, which give  $U_1 = w + \delta \overline{U}$  and  $U_0 = \overline{U}$ . Under binding (P-IC-e) and (P-IC0), equations (1) and (2) can be satisfied only if  $\frac{(1-\delta)\overline{U}}{c} = \frac{\alpha_0}{1-\alpha_1}$ . In that case, the solved *w* also coincides with

 $w_{DS}$ , and a buyer is thus indifferent among choosing any value of  $\beta \in [0, 1]$ .

**Case 4)**  $\mu_1 > 0$  and  $\mu_2 = 0$ . In this case,  $U_1 = w + \delta \overline{U}$ ,  $U_0 > \overline{U}$ ,  $\frac{\partial L}{\partial w} = 1 + \mu_1 (1 - \frac{\partial U_1}{\partial w}) = 0$ , and  $\frac{\partial L}{\partial \beta} = -(\mu_1 \frac{\partial U_1}{\partial \beta} + \mu_3) = 0$ . We discuss whether  $\beta = 1$  or not. If  $\beta \neq 1$ ,  $\mu_3 = 0$ , then  $\frac{\partial U_1}{\partial \beta} = 0$ . Given that, equation (A3) suggests that  $\frac{\partial U_0}{\partial \beta} < 0$ , while equation (A4) suggests that  $\frac{\partial U_0}{\partial \beta} > 0$ . Therefore, a contradiction exists, indicating that  $\beta = 1$ . Given  $\beta = 1$  and  $\mu_3 > 0$ ,  $w_{DI} = \left(\frac{1}{\delta} \frac{1 + \frac{\delta}{1 - \delta} \alpha_0}{(1 - \alpha_1) + \frac{\delta}{1 - \delta} \alpha_0}\right) c + \left(\frac{1 + \frac{\delta}{1 - \delta} \alpha_0}{(1 - \alpha_1) + \frac{\delta}{1 - \delta} \alpha_0} - \delta\right) \overline{U}$ .

In sum, if a buyer decides to choose  $\beta > 0$ , she will optimally choose  $\beta = 1$  with paying  $w_{DI}$  when her demand is 1. The comparison between choice 1 and choice 2 hinge on comparing  $w_{DS}$  and  $w_{DI}$ . It turns out that  $w_{DS} < w_{DI}$  if and only if  $\frac{(1-\delta)\overline{U}}{c} > \frac{\alpha_0}{1-\alpha_1}$ . Therefore, whenever the condition above holds, the buyer chooses  $\beta = 0$  (separating from the producer when demand becomes 0) and pays  $w_{DS}$  when her demand is 1. Otherwise, she chooses  $\beta = 1$  (retaining the producer when demand becomes 0) and pays  $w_{DI}$  when her demand is 1.

# Proof of Lemma 2

Observe that the continuation payoffs and incentive compatibility constraints for a manager in a managed contract are similar with those for a producer in a direct contract. The only differences are that the "cost of production" for a manager is  $w_M$  and its continuation value after separating from a buyer, given by  $\overline{V}$ , is 0. Since  $\frac{(1-\delta)\overline{V}}{w_M} \leq \frac{\alpha_0}{1-\alpha_1}$  for any  $\alpha_0$  and  $\alpha_1$ . Therefore, the value of p is determined by replacing c with  $w_M$  and  $\overline{U}$  with 0 in  $w_{DI}$ .

## **Proof of Proposition 1**

Step 1: Compare direct contract with separation and direct contract with idleness. From Lemma 1, it is immediate that if  $\overline{U} > \overline{U}^{ei} \equiv \frac{c}{1-\delta}\alpha_0$ , there exists a unique cutoff  $\overline{\alpha}_1^{ei} \in (0, 1)$  such that  $w_{DS} < w_{DI}$  if and only if  $\alpha_1 < \overline{\alpha}_1^{ei}$ .

Step 2: Compare direct contract with separation with managed contract. Observe that  $w_{DS} < p$  if and only if  $f(\alpha_1) \equiv \frac{1}{1-\alpha_1} \frac{c}{\delta} + (1-\delta)\overline{U} - \frac{1}{\delta - \frac{\delta}{1+\frac{\delta}{1-\delta}\alpha_0}\alpha_1} (\frac{c}{\delta} + (1-\delta)\overline{U}) < 0.$ 

Observe that, when  $\alpha_0 = 0$ ,  $f(\alpha_1)|_{\alpha_0=0} = (1 - 1/\delta)\frac{c}{\delta}\frac{1}{1-\alpha_1} + (1 - \frac{1}{\delta(1-\alpha_1)})(1-\delta)\overline{U} < 0$ . In this case,  $w_{DS} < p$  for sure, so  $\overline{\alpha}_1^{eo} = 1$ . When  $\alpha_0 > 0$ , observe that  $f(0) = (1 - \frac{1}{\delta})\kappa < 0$ , and f(1) goes to infinity. We now show that the continuous function  $f(\alpha_1)$  intersects with 0 only once. Note that  $\frac{\partial f(\alpha_1)}{\partial \alpha_1} = \frac{\zeta(\zeta \frac{c}{\delta} - \kappa)\alpha_1^2 - 2(c-\kappa)\zeta\alpha_1 + (\delta c - \zeta \kappa)}{(1-\alpha_1)^2(\delta - \zeta \alpha_1)^2}$ , where  $\zeta = \frac{\delta}{1+\frac{\delta}{1-\delta}\alpha_0}$  and  $\kappa = \frac{c}{\delta} + (1-\delta)\overline{U}$ . Observe that the numerator is a quadratic equation, where the coefficient of  $\alpha_1^2$  is negative, the coefficient of  $\alpha_1$  is positive, and the constant  $\delta c - \zeta(\frac{c}{\delta} + (1-\delta)\overline{U})$  can be positive or negative. Therefore,  $f(\alpha_1)$  is either strictly increasing, or is first decreasing then increasing. In either case,  $f(\alpha_1)$  intersects with 0, with the intersect being  $\overline{\alpha}_1^{eo} \in (0, 1)$ . In sum, there exists a unique cutoff  $\overline{\alpha}_1^{eo} \in (0, 1]$  such that  $w_{DS} < p$  if and only if  $\alpha_1 < \overline{\alpha}^{eo}$ .

Step 3: Compare direct contract with idleness and managed contract. Observe that  $w_{DI} < p$  if and only if  $g(\alpha_1) \equiv \frac{1+\frac{\delta}{1-\delta}\alpha_0}{1+\frac{\delta}{1-\delta}\alpha_0-\alpha_1} (\frac{2\delta-1}{\delta}\overline{U} - \frac{1-\delta}{\delta}\frac{c}{\delta}) < \delta\overline{U}$ . Observe that  $g(0) = \frac{2\delta-1}{\delta}\overline{U} - \frac{1-\delta}{\delta}\frac{c}{\delta}$  and  $g(1) = \frac{1+\frac{\delta}{1-\delta}\alpha_0}{\frac{\delta}{1-\delta}\alpha_0} (\frac{2\delta-1}{\delta}\overline{U} - \frac{1-\delta}{\delta}\frac{c}{\delta})$ . Also observe that  $g(0) - \delta\overline{U} = \frac{-(1-\delta)^2}{\delta}\overline{U} - \frac{1-\delta}{\delta}\frac{c}{\delta} < 0$ . If  $\overline{U} < \overline{U}^{io} \equiv \frac{(1-\delta(1-\alpha_0))c}{\delta(\delta(2-(1-\delta)\alpha_0)-1)}$ ,  $g(1) < \delta\overline{U}$  and thus the inequality holds for sure. Otherwise, since  $g(\alpha_1)$  is strictly increasing in  $\alpha_1$ , by the intermediate value theorem, there exists a unique cutoff  $\overline{\alpha}_1^{io} \in (0, 1)$  such that  $w_{DI} < p$  if and only if  $\alpha_1 < \overline{\alpha}_1^{io}$ .

**Step 4: Put it together.** Let  $\overline{U}^* = \overline{U}^{io}$  and  $\alpha^* = \min\{\overline{\alpha}_1^{io}, \overline{\alpha}_1^{eo}\}$ . If  $\overline{U} < \overline{U}^{io}$ ,  $w_{DI} < p$  by Step 3. Otherwise, if  $\overline{U} \ge \overline{U}^{io}$  there are two cases. If  $w_{DS} < w_{DI}$ , managed contract is optimal when  $p < w_{DS}$ , namely when  $\alpha_1 > \overline{\alpha}_1^{eo}$ . If  $w_{DS} > w_{DI}$  managed contract is optimal when  $p < w_{DI}$ , namely when  $\alpha_1 > \overline{\alpha}_1^{io}$ .

## **Proof of Corollary 1**

First, note that  $w_D(\alpha_i) > w_M$  for all  $i \in I_{DS} \cup I_{DI}$ . Therefore,  $E[w_D(\alpha_i) | i \in I_{DS} \cup I_{DI}] > E[w_M | i \in I_M]$ . Second, note that  $w_D(\alpha_i)$  takes on different values depending on  $\alpha_i$ , which is heterogeneous across buyers, while  $w_M$  is constant. Therefore,  $Var[w_D(\alpha_i) | i \in I_{DS} \cup I_{DI}] > Var[w_M | i \in I_M] = 0$ . Third, the separation rate is given by  $\beta_i \alpha_{1i}$ . Note that  $\beta_i = 1$  for all  $i \in I_{DI}$ , while  $\beta_i = 0$  for all  $i \in I_{DS} \cup I_M$ . Fourth, the idleness for a buyer  $i \in I_{DI}$  is given by  $(1 - \pi_i)(1 - \omega_i) > 0$ . By contrast, for  $i \in I_{DS} \cup I_M$ , idleness is zero.

# **Proof of Corollary 2**

With the introduction of managers, the per-period payoff of buyer  $i \in S_0$  increases by  $y - p(\alpha_i)$ . The per-period payoff of buyer  $i \in S_I$  increases by  $w_{DI}(\alpha_i) - p(\alpha_i)$ . The per-period payoff of buyer  $i \in S_S$  increases by  $w_{DS}(\alpha_i) - p(\alpha_i)$ . The per-period payoff of the remaining buyers are unchanged. The average producer pay per unit effort falls, since  $w_D(\alpha_i) > w_M$  for all  $i \in S_I \cup S_S$ . Producer separation rates and idleness also fall, since buyers switch from direct contracts with either idleness and separation to managed contracts in which buyers are never idle and never separate. The measure of buyers who receive services increases by  $|S_0|$ . The total measure of services provided in the economy increases by  $\int_{S_0} \pi_i dF$ . However, for  $i \in S_I$ , the steady-state measure of producers matched to these buyers fall, since idleness falls. The total measure of matched producer  $(n_M + n_B)$  therefore increases if and only if  $\int_{S_0} \pi_i dF > \int_{S_I} (1 - \pi_i) dF$ .

## **Proof of Proposition 3**

Based on Lemma 1 and 2, the buyer's post-matching continuation payoffs when she has demand is  $\Pi_D(\alpha_i) = \frac{y+\Delta_i}{1-\delta} - \left[\overline{U} + \frac{c}{(1-\alpha_i)\delta(1-\delta)}\right]$  if directly contracting with a specialist producer. It is  $\Pi_G = \frac{y}{1-\delta} - \left[\overline{U} + \frac{c}{\delta(1-\delta)}\right]$ , if directly contracting with a generalist producer. It is  $\Pi_M(\alpha_i) = \frac{y+\Delta_i}{1-\delta} - \left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{0}+\frac{1}{\delta}\alpha_i-\alpha_i}(\overline{U} + \frac{c}{\delta(1-\delta)})\right]$  if contracting with a manager.

Three observations follow. First, a buyer prefers to directly contract with a specialist than directly contract with a generalist if and only if  $\Pi_B(\alpha_i) \ge \Pi_G$ , or,  $\Delta_i \ge \frac{\alpha_i}{1-\alpha_i} \frac{c}{\delta}$ . Second, a buyer prefers to directly contract with specialists rather than contract with a manager if and only if  $\left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{1-\delta}\alpha_i-\alpha_i}-1\right]\left[\overline{U}+\frac{c}{\delta(1-\delta)}\right] \ge \frac{\alpha_i}{1-\alpha_i}\frac{c}{\delta(1-\delta)}$ . By Lemma **??**, there exists a unique cutoff  $\alpha^{ei}$  such that the buyer prefers to contract with a manager if and only if  $\alpha_i > \alpha^{ei}$ . Third, a buyer prefers a managed contract over direct contract with a generalist if and only if  $\Delta_i \ge \overline{\Delta}(\alpha_i) \equiv \left[\frac{1}{\delta}\frac{1+\frac{\delta}{1-\delta}\alpha_i}{1+\frac{\delta}{1-\delta}\alpha_i-\alpha_i}-1\right]\left[\overline{U}+\frac{c}{\delta(1-\delta)}\right]$ . Therefore, the buyer choose to contract with a manager if and only if  $\alpha_i$  and  $\Delta_i$  are both large enough.

# **Proof of Corollary 3**

Let  $S_G$  denote buyers who switch from direct contracts with generalists to managed contracts with specialists when managers become available. Let  $S_B$  be the set of buyers who switch from direct contracts with specialists to managed contracts with specialists. Given the assumption on the distribution of  $(\Delta_i, \alpha_i)$ ,  $|S_G|, |S_B| > 0$ . The contractual choice of the remaining buyers are unchanged. Therefore, the measure of specialists unambiguously increase with managerial coordination.

The Bellman equation for unmatched producers can be rewritten as  $\overline{U} = \frac{v}{u} \frac{E[v_i U_{1i}]}{v} + (1 - \frac{v}{u}) \delta \overline{U}$ . With the introduction of managers, more producers are contracted as specialists under managed contracts without separation, so *v* and  $E[v_i U_{1i}]$  both fall. This implies that  $\frac{v}{u}$  increases. It follows that the measure of unmatched producers  $n_N = u - v$  falls.

# **Proof of Proposition 4**

In a multi-task economy with reputable managers, the service fee is given by

$$p(\alpha_i) = \left(\frac{1}{\delta} \frac{1 + \frac{\delta}{1 - \delta} \alpha_i}{(1 - \alpha_i) + \frac{\delta}{1 - \delta} \alpha_i}\right) w_M + \left(\frac{1 + \frac{\delta}{1 - \delta} \alpha_i}{(1 - \alpha_i) + \frac{\delta}{1 - \delta} \alpha_i} - \delta\right) (-\gamma \tilde{V})$$

This implies that

$$\Pi_M(\alpha_i) = \frac{y + \Delta_i}{1 - \delta} - \left[\frac{1}{\delta} \frac{1 + \frac{\delta}{1 - \delta} \alpha_i}{1 + \frac{\delta}{1 - \delta} \alpha_i - \alpha_i} (\overline{U} + \frac{c}{\delta(1 - \delta)})\right] + \left(\frac{1 + \frac{\delta}{1 - \delta} \alpha_i}{(1 - \alpha_i) + \frac{\delta}{1 - \delta} \alpha_i} - \delta\right) \frac{\gamma \tilde{V}}{1 - \delta},$$

As  $\gamma$  increases,  $\Pi_M(\alpha_i)$  rises, so buyers switch to managed contracts with specialists from direct contracts with both generalists and specialists. Therefore, by the same logic as Corollary 3, the measure of specialists increases, while the measure of unmatched producers decreases.

# **Online Appendix**

# A Limited Liability, Stationarity, and Bonus Payments

In this appendix, we show that, in our model with limited liability, it is without loss of generality to focus on analyzing stationary relational contracts without bonus payments given  $\overline{U} > 0$ . To do so, we first show that, when the limited liability constraints are not imposed, it is without loss to focus on stationary relational contracts without bonus payments, following the idea in Sections A.1 and A.2 of Board and Meyer-ter-Vehn (2015). We then show that assuming limited liability is without loss.

Consider a model without limited liability where the buyer can pay the produce a bonus after observing the effort. In that model, suppose  $\beta_t > 0$ ,  $\forall t$ , the continuation payoffs of producers and buyers (omitting subscription *i*) are

$$\begin{split} U_{1,t} &= w_{1,t} - c(e_t) + b_{1,t} + \delta \Big[ (1 - \alpha_1) U_{1,t+1} + \alpha_1 (\beta_t U_{0,t+1} + (1 - \beta_t) \overline{U}) \Big], \\ U_{0,t} &= w_{0,t} + b_{0,t} + \delta \Big[ \alpha_0 U_{1,t+1} + (1 - \alpha_0) (\beta_t U_{0,t+1} + (1 - \beta_t) \overline{U}) \Big], \\ \Pi_{1,t} &= y(e_t) - w_{1,t} - b_{1,t} + \delta \Big[ (1 - \alpha_1) \Pi_{1,t+1} + \alpha_1 (\beta_t \Pi_{0,t+1} + (1 - \beta_t) \overline{\Pi}_0) \Big], \\ \Pi_{0,t} &= -w_{0,t} - b_{0,t} + \delta \Big[ \alpha_0 \Pi_{1,t+1} + (1 - \alpha_0) (\beta_t \Pi_{0,t+1} + (1 - \beta_t) \overline{\Pi}_0) \Big], \end{split}$$

respectively. Note that if  $\beta_{\tau} = 0$  for some  $\tau$ , there is no  $U_{0,t}$  and  $\Pi_{0,t}$  for  $t > \tau$ .

Suppose there exists a relational contract  $(e_t, w_{1,t}, b_{1,t}, w_{0,t}, b_{0,t}, \beta_t)_t$  that satisfies the following constraints:

$$U_{1,t} \ge w_{1,t} + \delta U,$$
  
 $U_{1,t} \ge \overline{U},$   
 $U_{0,t} \ge \overline{U},$ 

$$\Pi_{1,t} \ge \delta(\alpha_1 \Pi_0 + (1 - \alpha_1) \Pi_1)$$

$$\begin{split} \Pi_{1,t} \geq \overline{\Pi}_{1}, \\ \Pi_{0,t} \geq \overline{\Pi}_{0}, \\ -b_{1,t} + \delta \Big[ (1-\alpha_{1})\Pi_{1,t+1} + \alpha_{1}(\beta_{t}\Pi_{0,t+1} + (1-\beta_{t})\overline{\Pi}_{0}) \Big] \geq \delta(\alpha_{1}\overline{\Pi}_{0} + (1-\alpha_{1})\overline{\Pi}_{1}), \\ -b_{0,t} + \delta \Big[ \alpha_{0}\Pi_{1,t+1} + (1-\alpha_{0})(\beta_{t}\Pi_{0,t+1} + (1-\beta_{t})\overline{\Pi}_{0}) \Big] \geq \delta(\alpha_{0}\overline{\Pi}_{1} + (1-\alpha_{0})\overline{\Pi}_{1}). \end{split}$$

Compared to the constraints listed in Section 4, allowing bonus payments does not introduce any additional constraints for the producer because the original (P-IC-e) has captured the producer's incentive to exert efforts. The additional constraints for the buyer requires that the continuation values of keeping the producer whenever the producer has demand is higher than the bonus paid in either situation. Note again that if  $\beta_{\tau} = 0$  for some  $\tau$ , the constraints on  $\Pi_{0,t}$  does not exist for  $t > \tau$ .

# A.1 Irrelevance of bonus payments

Based on the relational contract with bonus payments  $(e_t, w_{1,t}, b_{1,t}, w_{0,t}, b_{0,t}, \beta_t)_t$ , we can construct a new relational contract without bonus payments that generates the same continuation payoffs for the buyer,  $(\hat{w}_{1,t}, \hat{w}_{0,t}, \hat{\beta}_t)_t$ , by shifting the bonus payments in the last period to the next period with adjusting to discounting and potential separation.

Specifically, let  $\hat{w}_{1,t} = w_{1,t} + \frac{(1-\alpha_0)\frac{b_1}{\delta} - \alpha_1\frac{b_0}{\delta}}{1-\alpha_0 - \alpha_1}$ ,  $\hat{w}_{0,t} = w_{0,t} + \frac{(1-\alpha_1)\frac{b_0}{\delta} - \alpha_0\frac{b_1}{\delta}}{1-\alpha_0 - \alpha_1}$ , and  $\hat{\beta}_t = \beta_t$ ,  $\forall t$ . By doing so, all the incentive constraints are satisfied since the following conditions are satisfied:

$$-b_{1,t} + \delta \Big[ (1-\alpha_1)\Pi_{1,t+1} + \alpha_1 (\beta_t \Pi_{0,t+1} + (1-\beta_t)\overline{\Pi}_0) \Big] = \delta \Big[ (1-\alpha_1)\hat{\Pi}_{1,t+1} + \alpha_1 (\hat{\beta}_t \hat{\Pi}_{0,t+1} + (1-\hat{\beta}_t)\overline{\Pi}_0) \Big]$$
$$-b_{0,t} + \delta \Big[ \alpha_0 \Pi_{1,t+1} + (1-\alpha_0) (\beta_t \Pi_{0,t+1} + (1-\beta_t)\overline{\Pi}_0) \Big] = \delta \Big[ \alpha_0 \hat{\Pi}_{1,t+1} + (1-\alpha_0) (\hat{\beta}_t \hat{\Pi}_{0,t+1} + (1-\hat{\beta}_t)\overline{\Pi}_0) \Big]$$

Under this relational contract without bonus payments, the buyer's continuation payoffs stay the same as the original relational contract with bonus payments. It is therefore without loss to focus on relational contracts without bonus payments.

# A.2 No loss from assuming stationarity

To show that it is without loss of generality to focus on stationary relational contracts, we now construct a stationary relational contract  $(e^*, w_1^*, w_0^*, \beta^*)$  based on any potentially nonstationary relational contract  $(e_t, w_{1,t}, w_{0,t}, \beta_t)_t$  that generates a weakly higher continuation payoff for the buyer when starting a relationship, i.e.,  $\overline{\Pi}_1^* \ge \overline{\Pi}_1$ .

Under  $(e_t, w_{1,t}, w_{0,t}, \beta_t)_t$ , the producer's and the buyer's continuation payoffs in a match are

$$\begin{split} U_{1,t} &= w_{1,t} - c(e_t) + \delta \Big[ (1 - \alpha_1) U_{1,t+1} + \alpha_1 (\beta_t U_{0,t+1} + (1 - \beta_t) \overline{U}) \Big], \\ U_{0,t} &= w_{0,t} + \delta \Big[ \alpha_0 U_{1,t+1} + (1 - \alpha_0) (\beta_t U_{0,t+1} + (1 - \beta_t) \overline{U}) \Big], \\ \Pi_{1,t} &= y(e_t) - w_{1,t} + \delta \Big[ (1 - \alpha_1) \Pi_{1,t+1} + \alpha_1 (\beta_t \Pi_{0,t+1} + (1 - \beta_t) \overline{\Pi}_0) \Big], \\ \Pi_{0,t} &= -w_{0,t} + \delta \Big[ \alpha_0 \Pi_{1,t+1} + (1 - \alpha_0) (\beta_t \Pi_{0,t+1} + (1 - \beta_t) \overline{\Pi}_0) \Big]. \end{split}$$

The incentive constraints for the producer and the buyer are:

$$U_{1,t} \ge w_{1,t} + \delta \overline{U},$$
$$U_{1,t} \ge \overline{U},$$
$$U_{0,t} \ge \overline{U},$$
$$\Pi_{1,t} \ge \delta(\alpha_1 \overline{\Pi}_0 + (1 - \alpha_1) \overline{\Pi}_1)$$
$$\Pi_{1,t} \ge \overline{\Pi}_1,$$
$$\Pi_{0,t} \ge \overline{\Pi}_0.$$

Now we construct  $(e^*, w_1^*, w_0^*, \beta^*)$ . First, suppose  $e_{t'} = 1$  in some period t' and let  $e^* = e_{t'} = 1$  (t' exists since otherwise the original relational contract will be a stationary in the effort with effort being 0 in all periods). Second, let  $U_1^* = U_{1,t'}$  and  $U_0^* = U_{0,t'}$ . Finally,

let  $\beta^* = \beta_{t'}$  and set the payments  $w_1^*$  and  $w_0^*$  such that they satisfy:

$$U_1^* = w_1^* - c + \delta \Big[ (1 - \alpha_1) U_1^* + \alpha_1 (\beta^* U_0^* + (1 - \beta^*) \overline{U}) \Big],$$
$$U_0^* = w_0^* + \delta \Big[ \alpha_0 U_1^* + (1 - \alpha_0) (\beta^* U_0^* + (1 - \beta^*) \overline{U}) \Big].$$

The buyer's continuation payoffs are thus

$$\Pi_1^* = y - w_1^* + \delta \Big[ (1 - \alpha_1) \Pi_1^* + \alpha_1 (\beta^* \Pi_0^* + (1 - \beta^*) \overline{\Pi}_0) \Big],$$
$$\Pi_0^* = -w_0^* + \delta \Big[ \alpha_0 \Pi_1^* + (1 - \alpha_0) (\beta^* \Pi_0^* + (1 - \beta^*) \overline{\Pi}_0) \Big].$$

Under this new stationary relational contract, the producer's incentive constraints are satisfied. Indeed, the first incentive constraint requires

$$\delta\left[(1-\alpha_1)(U_1^*-\overline{U})+\alpha_1\beta^*(U_0^*-\overline{U})\right]\geq c,$$

as  $\delta \left[ (1 - \alpha_1)(U_{1,t'} - \overline{U}) + \alpha_1 \beta_{t'}(U_{0,t'} - \overline{U}) \right] \ge c(e_{t'})$  at t'. Meanwhile, we know  $U_1^* = U_{1,t'} \ge \overline{U},$  $U_0^* = U_{0,t'} \ge \overline{U},$ 

The buyer's incentive constraints are satisfied since the relational contract is stationary, which implies  $\Pi_1^* = \overline{\Pi}_1$  and  $\Pi_0^* = \overline{\Pi}_0$  in this frictionless market with excess supply of producers.

To show  $\overline{\Pi}_1^* \ge \overline{\Pi}_1$ , given that  $\overline{\Pi}_1^* = \Pi_1^*$  and  $\Pi_{1,t'} \ge \overline{\Pi}_1$ , we need only to show  $\Pi_1^* \ge \Pi_{1,t'}$ . We do so by comparing the joint surpluses  $\Pi_1^* + U_1^*$  and  $\Pi_{1,t'} + U_{1,t'}$ . It is obvious that  $\Pi_1^* + U_1^* \ge \Pi_{1,t'} + U_{1,t'}$ , since under the stationary relational contract, effort is motivated and thus the flow payoffs are maximized in each period when there is demand. Given that  $U_1^* = U_{1,t'}$  by construction, we have  $\Pi_1^* \ge \Pi_{1,t'}$ .

# A.3 No loss from assuming limited liability

We now show that imposing limited liability is without loss. The idea is that, if the limited liability constraints are violated under an equilibrium relational contract (i.e., if the pay is below zero when there is or isn't demand), we can show that either 1) there exists another equilibrium relational contract where the limited liability constraints are not violated, or 2) the original relational contract cannot be sustained in equilibrium.

Under a stationary relational contract without bonus payments  $(w_1, w_0, \beta)$  (omitting effort as it is always 1), the continuation payoffs are

$$\begin{split} U_{1} &= w_{1} - c + \delta \Big[ (1 - \alpha_{1})U_{1} + \alpha_{1}(\beta U_{0} + (1 - \beta)\overline{U}) \Big], \\ U_{0} &= w_{0} + \delta \Big[ \alpha_{0}U_{1} + (1 - \alpha_{0})(\beta U_{0} + (1 - \beta)\overline{U}) \Big], \\ \Pi_{1} &= y - w_{1} + \delta \Big[ (1 - \alpha_{1})\Pi_{1} + \alpha_{1}(\beta\Pi_{0} + (1 - \beta)\overline{\Pi}_{0}) \Big], \\ \Pi_{0} &= -w_{0} + \delta \Big[ \alpha_{0}\Pi_{1} + (1 - \alpha_{0})(\beta\Pi_{0} + (1 - \beta)\overline{\Pi}_{0}) \Big]. \end{split}$$

The incentive constraints for the producer and the buyer are

$$U_{1} \ge w_{1} + \delta U,$$

$$U_{1} \ge \overline{U},$$

$$U_{0} \ge \overline{U},$$

$$\Pi_{1} \ge \delta(\alpha_{1}\overline{\Pi}_{0} + (1 - \alpha_{1})\overline{\Pi}_{1}),$$

$$\Pi_{1} \ge \overline{\Pi}_{1},$$

$$\Pi_{0} \ge \overline{\Pi}_{0}.$$

The limited liability constraints are

$$w_1 \ge 0,$$

$$w_0 \ge 0.$$

Note that we can solve

$$\Delta U_1 = \frac{w_1 - c - (1 - \delta)\overline{U}}{1 - \delta(1 - \alpha_1)} + \frac{\delta \alpha_1 \beta}{1 - \delta(1 - \alpha_1)} \Delta U_0,$$

$$\Delta U_0 = \frac{\delta \alpha_0 (w_1 - c - (1 - \delta)U) + (1 - \delta(1 - \alpha_1))(w_0 - (1 - \delta)U)}{(1 - \delta(1 - \alpha_1))(1 - \delta(1 - \alpha_0)\beta) - \delta^2 \alpha_0 \alpha_1 \beta}$$

where  $\Delta U_1 = U_1 - \overline{U}$  and  $\Delta U_0 = U_0 - \overline{U}$ . Further note that, from Lemma A.1,  $w_0 > 0$  cannot be sustained in equilibrium. Therefore, there are three cases where the limited liability constraints can be violated.

The first case is that both limited liability constraints are violated, i.e.,  $w_1 < 0$  and  $w_0 < 0$ . In this case, both  $\Delta U_1$  and  $\Delta U_0$  are strictly negative. The producer's incentive constraints are thus violated. It is thus without loss not to consider such a relational contract as it cannot be sustained in equilibrium.

The second case is that only the limited liability constraint on  $w_1$  is violated, i.e.,  $w_1 < 0$ and  $w_0 = 0$ . In this case, both  $\Delta U_1$  and  $\Delta U_0$  are still strictly negative, indicating that the producer's incentive constraints are violated. Again, it is thus without loss not to consider such a relational contract as it cannot be sustained in equilibrium.

The third case is that only the limited liability constraint on  $w_0$  is violated, i.e.,  $w_1 \ge 0$  and  $w_0 < 0$ . In this case, we increase  $w_0$  and decrease  $w_1$  such that  $w_0$  can equal to 0. Specifically, based on  $(w_1, w_0, \beta)$ , we construct a new relational contract  $(w_1^*, w_0^*, \beta^*)$  by setting  $w_0^* = 0$  while keeping  $U_1^* = U_1$ . To do so we set  $w_1^* = w_1 + \frac{\delta \alpha_1 \beta}{1 - \delta(1 - \alpha_0)\beta} w_0$ . By construction,  $U_0^* = U_0 - \frac{w_0}{1 - \delta(1 - \alpha_0)\beta} > U_0$  given  $w_0 < 0$ . Under this new relational contract, the producer's incentive constraints are satisfied since his continuation payoffs weakly increase, and the

buyer's incentive constraints are satisfied since  $\Pi_1$  stays the same and  $\Pi_0^* \ge \overline{\Pi}_0^*$ . The latter comes from the fact that  $\Pi_0^* - \overline{\Pi}_0^* = \frac{-w_0^*}{1-\delta(1-\alpha_0)\beta} = 0$  given  $\Pi_1^* = \Pi_1 = \overline{\Pi}_1 = \overline{\Pi}_1^*$  and  $w_0^* = 0$ . Further, the buyer is indifferent between the original and the newly constructed relational contracts, as  $\Pi_1^* = \Pi_1$ . The last step is to check whether  $w_1^* \ge 0$ . If not, we have  $w_1^* < 0$ and  $w_0^* = 0$ , which is the same as the second case above and thus cannot be sustained in equilibrium. Therefore, should the original relational contract be sustained in equilibrium,  $w_1^* \ge 0$ , indicating both the limited liability constraints are satisfied.

In sum, imposing limited liability constraints is without loss.

# **B** Exogenous Producer Population

The main text assumed that the number of producers is determined via endogenous entry. In this appendix, we analyze an economy where the number of producers is fixed exogenously at n > 1. The main difference is that when the producer population is fixed, the presence of managers alters  $\overline{U}$  by changing the overall demand for producers, and thereby alters equilibrium outcomes.

We first show that if *n* is sufficiently large, then the producer market becomes very slack, buyers prefer to enter contracts with idleness, and there is no managerial coordination.

**Proposition B.1.** Assume the number of producers n is exogenously fixed. There exists a unique steady-state equilibrium. If n is sufficiently large, then there are no managed contracts.

*Proof.* Given  $\overline{U}$ , we can compare the values of  $w_{DS}(\alpha_i)$ ,  $w_{DI}(\alpha_i)$ ,  $p(\alpha_i)$ , and y for each *i* using (4), (5), and (9) to determine  $I_{DS}$ ,  $I_{DI}$ , and  $I_M$ . Having derived these, we can obtain unique values for  $v_i$ , v,  $n_B$ , and  $n_M$  from (10), (11), (12), and (13). Plugging  $w_{DS}(\alpha_i)$ ,  $w_{DI}(\alpha_i)$ , and  $w_M(\alpha_i)$  into (16) yields a contraction mapping. Therefore a unique fixed point for  $\overline{U}$  exists. By (14), (15), and (16),  $\overline{U}$  is decreasing in n, and  $\overline{U} \to 0$  as  $n \to \infty$ . The desired result follows by Proposition 1.

We next compare economies with and without managers assuming that *n* is fixed and that some nonzero measure of buyers choose managerial coordination when possible. In this economy, introducing managers has both direct and indirect effects. The direct effect holds  $\overline{U}$  fixed and was characterized in the previous subsection. Due to this effect, various types of buyers switch to managerial coordination, producer rents fall, and the number of matched producers may or may not increase. Without free entry, however,  $\overline{U}$  also adjusts. A change in  $\overline{U}$  affects all of the endogenous variables in the model, so managerial coordination has indirect spillover effects onto all producers.

The sign of the resulting change in  $\overline{U}$  depends on the distribution of buyer types  $\alpha_i$ . If a large set of buyers switch from being unmatched to matched, then  $\overline{U}$  would increase due to reduced wait time in a tightened producer market. This indirect effect raises the pay of all producers. On the other hand, if no buyers switch from being unmatched to being matched, then  $\overline{U}$  would unambiguously fall due to producer rent reductions associated with initially matched buyers switching to managerial coordination. This indirect effect instead reduces the pay of all producers.