Housing Expenditure and Misallocation in a Rent Regulated City

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Abstract

We present new theory and evidence on the equilibrium effects of rent regulation. Departing from existing models, we develop a framework showing that rent regulation may generate divergent price effects on high- and low-quality unregulated units, as housing becomes misallocated across income groups. The model explains the recent evolution of the housing market in Hong Kong—a global metropolis where nearly one-third of residents occupy rent-regulated public housing. Leveraging detailed population census data from 2006 to 2016, we document increasing misallocation of public housing to higher-income tenants over time. Concurrently, rent growth in lower-quality private-sector units significantly outpaced rent growth in higher-quality segments. Our analysis reveals that although rent regulation shielded incumbent public tenants from market pressures, it exacerbated affordability challenges for younger generations and migrants.

Keywords: rent control, housing expenditure, misallocation
JEL: L33, O18, P25, R31, R38

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1 Introduction

Economists and policymakers increasingly warn that the lack of affordable housing in productive urban centers stifles economic growth and deepens inequality (Glaeser and Gyourko 2018; Hsieh and Moretti 2019; Dustmann, Fitzenberger and Zimmermann 2022; Baum-Snow 2023). To ensure an adequate supply of affordable housing, many governments have imposed rent control. Others have increased the provision of rent-regulated public housing.

However, it is uncertain whether rent regulation can mitigate the negative aggregate consequences of insufficient affordable housing. Some scholars find that rent control depresses the average prices of nearby uncontrolled homes (Wang 2011; Autor, Palmer and Pathak 2014). Evidence from Germany and elsewhere instead suggests that rent control increases average rents outside the controlled sector (Mense, Michelsen and Kholodilin 2023; Kholodilin 2024). A clear picture of the equilibrium effects of rent regulation has yet to emerge.

In this paper, we offer new theory and evidence on the equilibrium effects of rent regulation. First, we relax the assumption in existing models of rent control that houses are perfectly divisible (Gould and Henry 1967; Fallis and Smith 1984; Wang 2011). We show that rent regulation can have opposite effects on the equilibrium rents of low-quality and high-quality units, leading to harmful consequences for low-income renters outside the regulated sector. Second, we disaggregate recent trends in housing expenditures, allocation, rents, price, and supply in Hong Kong—a metro area with rising housing demand and significant rent regulation. Consistent with our theory, our findings suggest that rent regulation exacerbated rather than alleviated some of the negative effects of inadequate affordable housing. Although incumbent households were protected from rising rents, burdens on low-income migrants and younger generations increased, likely with detrimental impacts on city growth.

The theory of rent control proposed here builds on a growing literature that uses assignment models to analyze housing markets (e.g., Määttänen and Terviö 2014; Landvoigt, Piazzesi and Schneider 2015; Epple, Quintero and Sieg 2020). In addition to introducing rent control, a novelty here is a flexible specification of supply substitution patterns across housing quality segments. In our model, low-quality rent-controlled housing is rationed among interested households. In the uncontrolled sector, a set of developers competitively decide the quantities of houses of either quality to build. Landlords of existing rental housing stock choose whether to alter the quality of houses through subdivision or combination. Households sort themselves into different quality segments according to their income levels. Rents are competitively set so that the housing markets clear.

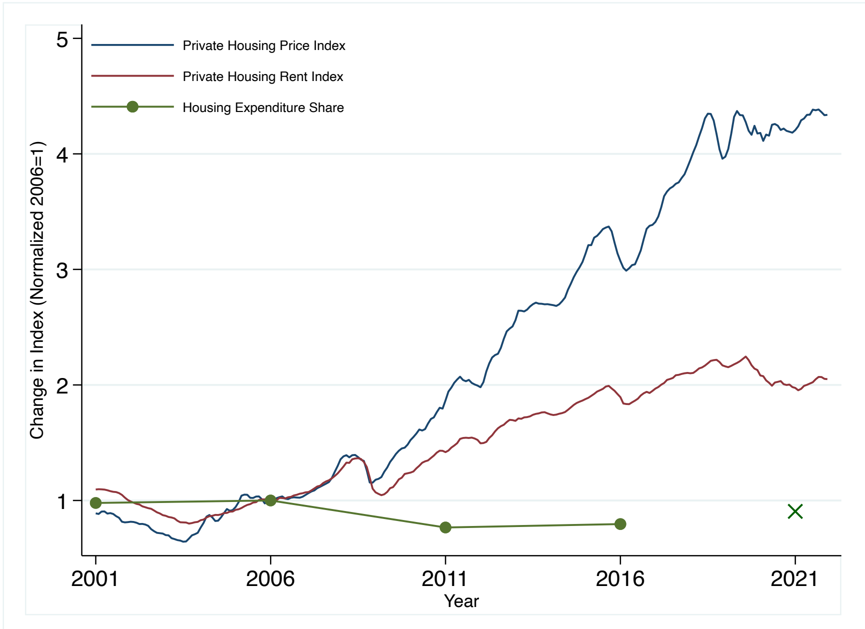
We characterize the effects of two thought experiments: (1) a reduction in controlled rent, and (2) a rent control expansion that increases the share of controlled housing. In each case, the exogenous change alters the degree of housing misallocation, which in turn affects demand in the uncontrolled sector and influences equilibrium rents. To illustrate how rent control may have opposite effects on rents in different quality segments, we focus on the case where the quality of existing housing stock is difficult to transform.

In the first experiment, lowering the controlled rent encourages higher-income households, who would otherwise rent high-quality housing, to apply for controlled units. This increases competition for rent-controlled housing, reduces the acceptance rate, and pushes more rejected applicants into the uncontrolled low-quality segment. Consequently, the relative demand for low-quality uncontrolled housing rises, driving up its rent, while demand for high-quality housing falls, lowering its rent.

In the second experiment, expanding rent control mechanically reduces the supply of uncontrolled low-quality housing while shifting demand away from both high-quality and low-quality uncontrolled units due to increased acceptance rates. Since some of the newly occupied rent-controlled units are taken by households who would have rented high-quality housing, excess demand arises for low-quality uncontrolled units, while high-quality units experience excess supply. In response, the housing supply system adjusts by increasing the supply of low-quality housing and decreasing that of high-quality housing, causing rents in the two segments to move in opposite directions.

We provide evidence that is consistent with our proposed theory. Our empirical context—Hong Kong—is a particularly interesting setting for studying housing affordability and rent regulation for three reasons. First, a large fraction of its population lives in public rental housing with below-market rents. Second, unlike many other settings, Hong Kong's population data record housing expenditures and separately identify public- and private-sector residents. Third, Hong Kong has been ranked the least affordable housing market in the world every

Figure 1: Trends in price, rent, and housing expenditure shares, 2001-2021



Notes: Figure plots the change in private housing price and rent indexes and housing expenditure share. The housing expenditure share is calculated as the ratio of equivalised monthly housing expenditure over equivalised monthly household income, where household income is defined as the sum of earnings in cash from all employment and other cash income, and housing expenditure for renters includes basic rent, while housing expenditure for owner-occupiers includes mortgage payment. Because of the inconsistent household income definition, the housing expenditure share in 2021 is not comparable with other waves. The index was normalized in 2006 to one. *Source: Rating and Valuation Department; Hong Kong Census Data 2001 - 2021*

year since 2010 (Cox 2024).

The years 2006 and 2016 are chosen as our focus, so that our comparison over time is not skewed by changes in income definitions in the Population Census data, major changes in housing policy, or short-term distortions such as the Covid-19 pandemic. During this period, Hong Kong experienced slow urban development and rapid increases in housing prices and rents. As shown in Figure 1, Hong Kong's private housing price index increased by 241% and its private rent index increased by 94% between 2006 and 2016.

We first document that public housing rent regulation significantly distorted housing expenditures in Hong Kong, even in our baseline year, 2006. Throughout the income distribution, public-sector residents spent much less as a fraction of their income than similar private-sector counterparts. Consequently, the relationship between housing expenditure shares and income exhibits an unusual U-shaped relationship: Households in the lowest-income decile spent 21% of their income on housing, households in the fourth and fifth deciles spent 12%, and the highest decile households spent 16%.

We then assess the impact of rising housing demand in the presence of rent regulation by decomposing the change in housing outcomes between 2006 and 2016. Changes in housing expenditure shares are decomposed by income, housing tenure, and demographics. Changes in housing tenure are broken down by income group and age cohort. Trends in housing price, rent, and supply are disaggregated by quality segment. Three main findings emerge.

First, public housing rent regulation shielded much of the population from rising private-sector housing prices and rents. Average housing expenditure as a share of income *fell* from a relatively low 15% in 2006 to an even lower 12% in 2016, even though housing prices and rents skyrocketed. Instead, the only group that experienced increases in housing expenditure shares was private renters, who accounted for only 11 percent of the population in 2006. Meanwhile, the average housing expenditure share of homeowners and public renters fell.

Second, public rental housing became increasingly misallocated towards middle-income households. The share of population living as public renters in the middle income quintile increased by about 4 percentage points. At the same time, the share of public renters in the lowest income quintile fell by more than 5 percentage points. We also document that younger age cohorts increasingly lived with parents and rented in the private sector, and they became much less likely to become owners or public renters.

Third, the prices, rents, and supply of low-quality housing disproportionately increased. The rents of private-sector units of similar quality to public rental housing more than doubled, while the rents of higher-quality private-sector units increased by less than one-half. The construction of small units dramatically increased. These findings suggest that there was disproportionate growth in the demand for private rental units in the lower-quality segment.

While the above evidence confirms the idea in existing models that rationing of controlled housing leads to housing misallocation across the income distribution (e.g., Glaeser and Luttmer 2003), our final finding contradicts the assumption in existing literature that houses in the uncontrolled sector are perfectly divisible and have a single rent level (e.g., Wang 2011). Our empirical findings therefore confirm the need for models in which rent control may have heterogeneous effects on the rents of uncontrolled housing in different quality segments.

The housing assignment model developed here provides a useful lens for understanding the heterogeneous equilibrium impacts of rent regulation across housing market segments.

1.1 Related Literature and Contributions

To our knowledge, this paper is the first to study rent control using a model with imperfectly divisible houses. In an early contribution, Gould and Henry (1967) developed a general equilibrium model to demonstrate that price controls can either raise or lower the price of a substitute good. Fallis and Smith (1984) argued that the impact of rent control on prices in the uncontrolled sector depends on the allocation mechanism in the controlled sector. Wang (2011) provides theoretical results that the effect of rent control on prices in the uncontrolled sector depends on the nature of misallocation. All of these theoretical contributions assume that uncontrolled houses are perfectly divisible and have a single price per housing service. Our model of rent control relaxes this unrealistic assumption.

Our theory builds on and extends a growing literature that uses assignment models to understand housing markets (Määttänen and Terviö 2014; Landvoigt, Piazzesi and Schneider 2015; Epple, Quintero and Sieg 2020). To this literature, we contribute evidence for the relevance of housing assignment models for analyzing the effects of rent control. We also introduce a novel and tractable approach to modeling endogenous housing supply that allows for flexible substitution patterns. In related work, Epple, Quintero and Sieg (2020) provide a quantitative model with endogenous supply, but assume no substitution across housing quality segments. Kang (2023) studies the effects of public options on equilibrium prices and shows that rationing may be welfare-optimal, but assumes that goods of different quality are created in fixed proportions from a single input. In both cases, the range of substitution patterns that can arise is highly limited.

Our evidence adds to an empirical literature on the misallocation and spillover effects of rent control. Existing evidence ignores the latter possibility and instead focuses on the impact of rent control on *average* rents. These studies have yielded conflicting results. For example, Wang (2011) documents that state misalloction of housing in China led the average prices of uncontrolled housing to fall. Autor, Palmer and Pathak (2014) show that rent control in Cambridge, Massachusetts lowered the average prices of both rent controlled and nearby houses. Yet, Mense, Michelsen and Kholodilin (2023) present quasi-experimental evidence from Germany that rent control raised average uncontrolled rent instead. We contribute evidence that rising private housing rents was associated with increased housing misallocation. We also find a disproportionate increase in rents in low-quality housing segments.

This paper relates to a growing literature on housing expenditure and inequality. Recent studies have consistently documented a widening gap in housing expenditures between renters and owners as well as between regions (e.g., Dustmann, Fitzenberger and Zimmermann 2022). To date, however, few scholars have considered the potential impact of rent regulation on inequality within a city. Our study contributes novel evidence that large-scale rent regulation can dramatically alter the patterns of housing consumption across the income distribution.

Finally, our evidence relates to a growing literature on urban housing affordability (Sieg and Yoon 2020; Favilukis, Mabille and Van Nieuwerburgh 2023). In the Hong Kong context, prior studies find that trends in Hong Kong's housing prices have a weak relationship with factors that drive housing demand, including credit conditions, economic growth, and Tobin taxes (Leung, Ng and Tang 2020*b*; Taghizadeh-Hesary et al. 2020; Agarwal et al. Forthcoming). Some scholars therefore suggest that a limited supply of land explains Hong Kong's high housing prices (Leung, Ng and Tang 2020*a*). This paper shows instead that public housing rent regulation is a key mediator of housing inequality and dynamics in Hong Kong, a factor that has been largely ignored in existing studies on the relevant period.

The paper proceeds as follows. Section 2 develops our theory of rent control. Section 3 provides background on Hong Kong's housing market. Section 4 documents the evolution of housing expenditures, misallocation, rents, and supply in Hong Kong between 2006 and 2016. Section 5 concludes.

2 Model of Housing Assignment with Rent Control

This section develops our theory of rent control. Unlike prior models (e.g. Wang 2011), we do not assume that houses are perfectly divisible. In our model, households sort one-to-one into houses, and transforming houses across quality segments requires convex adjustment costs. We show that rent control expansion and a decline in controlled rents not only increase housing misallocation, but can also cause uncontrolled rents in different quality segments to move in opposite directions.

2.1 Setup

There is a measure n of households. Each household derives utility $u(c, j)$ from consuming a house of quality j and non-housing consumption c, where $u_c, u_j, u_{cj} > 0$ and $u_{cc} < 0$. Household incomes follow a distribution $I \sim F$ with support on $[\underline{I}, \overline{I}]$, where $\overline{I} > \underline{I} > 0$. Household choices must satisfy $c + r \leq I$, where r denotes rent. The price of non-housing consumption is normalized to one. There are two discrete house quality levels, denoted by $j \in \{H, L\}$, where $H > L$.

The initial stock of uncontrolled houses of each quality is denoted by (h_L^0, h_H^0). The rents of uncontrolled houses are determined in market equilibrium, and are denoted as r_j for each quality j. The remaining housing stock is L-quality housing subject to rent control, with an exogenously set rent of $r_C < r_L$. The supply of these controlled houses is fixed at a constant share μ in total houses.

The final supply of uncontrolled houses is endogenous and consists of two parts. First, a representative developer supplies new uncontrolled houses of each quality, denoted as (h_L^n, h_H^n). Constructing a new house of quality j requires a fixed cost c_j and θ_j units of input k, such as land, which is priced at p. The developer solves the following profit maximization problem:

$$\max_{h_H^n, h_L^n} \quad (r_H - c_H - p\theta_H)h_H^n + (r_L - c_L - p\theta_L)h_L^n. \tag{1}$$

Second, a representative landlord can transform the initial stock of uncontrolled houses. The final supply of uncontrolled houses of each quality is denoted by (h_L^l, h_H^l). We assume that the landlord solves the following profit maximization problem:

$$\begin{aligned} &\max_{h_H^l, h_L^l} r_H h_H^l + r_L h_L^l - \phi\left(h_L^l - h_L^0\right) \\ &\text{s.t.} \quad h_H^l - h_H^0 = -\frac{\theta_L}{\theta_H}(h_L^l - h_L^0). \end{aligned} \tag{2}$$

The net quantity of L-quality houses created through transformation, $h_L^l - h_L^0$, can be either positive or negative. Each unit of L-quality house can be transformed into $\frac{\theta_L}{\theta_H}$ units of H-quality house, and *vice versa*. The landlord incurs an adjustment cost $\phi(h_L^l - h_L^0)$ for transformations, where $\phi'' > 0$, and $\phi(0) = \phi'(0) = 0$.

The total supply of uncontrolled houses is $h_j(r_L, r_H) \equiv h_j^l(r_L, r_H) + h_j^n(r_L, r_H)$ for $j \in \{L, H\}$. The induced demand for inputs is $k = \theta_H h_H^n + \theta_L h_L^n$. The input price p is determined by market clearing, with an inverse supply function $p(k)$, where $p' > 0$.

The timing of the model is as follows. First, the rent control share of housing μ is chosen. Second, all households who are willing to live in a rent-controlled house apply and are randomly assigned with equal probability. Third, the uncontrolled rent for each house quality and the input price adjust such that: (1) the choices of unassigned households maximize utility given prevailing prices, (2) the choices of landlord and developers maximize their profit given prevailing rents and input prices, and (3) all markets clear.

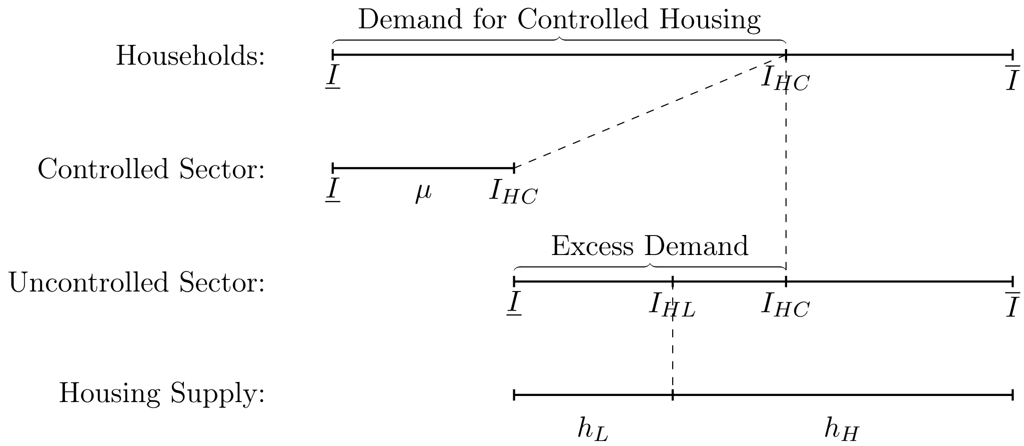
Remark. Several simplifying assumptions are made above for tractability. First, rent-controlled units are randomly rationed. Alternative rationing mechanisms can be used without altering the model's basic insights. Second, there are only two discrete levels of housing quality. This assumption allows us to introduce flexible substitution patterns on the supply side, which are not easily added into models where housing quality is instead continuous (e.g. Kang (2023)). Third, only low-type houses are rent controlled. This assumption is realistic, since in Hong Kong and many empirical settings, rent controlled homes are predominantly of lower quality.

2.2 Equilibrium

Due to utility maximization and the single crossing property of u, households will apply for rent-controlled houses if and only if their incomes are below a threshold I_{HC}. This threshold is the income of a household who is indifferent between renting a rent-controlled house and a high-quality house, and is defined by the following indifference condition:

$$u(I_{HC} - r_H, H) = u(I_{HC} - r_C, L). \tag{3}$$

Figure 2: Equilibrium rationing and sorting



Random rationing implies that only a fraction of the households who apply to rent in the controlled sector receive rent-controlled houses. The fraction of households wishing to enter the rent-controlled sector is $F(I_{HC})$, while the share of controlled houses is μ. We assume that $r_C < r_L$, so that $\mu < F(I_{HC})$. The acceptance rate is thus $\mu/F(I_{HC}) < 1$.

In the uncontrolled sector, competitive allocation implies that there exists an income I_{HL} such all households with income $I > I_{HL}$ reside in H-type housing and the remainder live in L-type housing. The household with income I_{HL} must be indifferent between H and L housing, and their indifference condition is:

$$u(I_{HL} - r_H, H) = u(I_{HL} - r_L, L). \tag{4}$$

Figure 2 illustrates the rationing of controlled houses and the sorting of households into uncontrolled housing segments in our model.

Market clearing for the two types of housing implies that:

$$\left(1 - \frac{\mu}{F(I_{HC})}\right) nF(I_{HL}) = h_L(r_L, r_H), \tag{5}$$

and

$$h_H(r_L, r_H) + h_L(r_L, r_H) = n(1 - \mu). \tag{6}$$

Together, Equations (3), (4), (5), and (6) determine the endogenous quantities r_L, r_H, I_{HC}, and I_{HL} as functions of the exogenous parameters μ and r_C.

To derive our main results, we make two assumptions throughout the analysis below.

Assumption 1. *Developers produce only H-type housing in equilibrium.*

The developer's problem in Equation (1) implies that if $r_j < c_j + p\theta_j$, the developer produces zero j-quality houses, so her supply of a given quality of housing is completely inelastic. If instead $r_j = c_j + p\theta_j$, then she may produce any nonnegative quantity, so her supply of housing is determined by the input market clearing condition. Assumption 1 implicitly states that housing production technology is such that $p = \frac{r_H - c_H}{\theta_H} > \frac{r_L - c_L}{\theta_L}$, so the developer produces a positive quantity of H-type housing and no L-type housing. Since new construction is predominantly of higher quality in many cities, this assumption is realistic.

The combined uncontrolled housing supply functions can then be written as:

$$h_L(r_L, r_H) = h_L^0 + \phi'^{-1}\left(r_L - \frac{\theta_L}{\theta_H} r_H\right), \tag{7}$$

and

$$h_H(r_L, r_H) = h_H^0 + \frac{1}{\theta_H} p^{-1}\left(\frac{r_H - c_H}{\theta_H}\right) - \frac{\theta_L}{\theta_H}\phi'^{-1}\left(r_L - \frac{\theta_L}{\theta_H} r_H\right). \tag{8}$$

Assumption 2. *$\phi'' > \theta_L (\theta_H - \theta_L) p'$.*

Assumption 2 states that the convexity in the housing transformation cost function of landlords is large relative to the slope of the developer input supply curve. In other words, it is difficult for landlords to significantly transform the quality composition of existing housing. Together, Assumptions 1 and 2 imply that the total supply of houses increases with respect to both r_H and r_L, holding the other fixed. As shown below, Assumptions 1 and 2 are necessary and sufficient conditions for rent regulation to have opposite effects on the rents of high-quality and low-quality uncontrolled houses.

For added intuition, we will focus on a special case for part of the analysis below. Cobb-Douglas utility, $u(c, j) = c^{\alpha} j^{1-\alpha}$, and concave CDF of income, such as uniform or Pareto. Under these assumptions, Equations (3) can be rewritten as $I_{HC} = (1+\sigma)r_H - \sigma r_C$. Equations

(4) can be then rewritten as $I_{HL} = (1+\sigma)r_H - \sigma r_L$, where $\sigma \equiv \frac{(L/H)^{\frac{1-\alpha}{\alpha}}}{1-(L/H)^{\frac{1-\alpha}{\alpha}}} > 0$. These linear equations significantly increase tractability. We call this special case the *linearized* model.

2.3 Effects of Change in Controlled Rent

We now consider how a reduction in the controlled rent r_C affects rents in the two uncontrolled sectors. We show that the change increases L-quality uncontrolled housing rents, even as it reduces H-quality uncontrolled housing rents.

Proposition 1. *The equilibrium rents of L-quality houses decrease in the controlled rent, while the equilibrium rents of H-quality houses increase in the controlled rent.*

Proof. See Appendix A.1. □

Proposition 1 is formally shown by totally differentiating Equations (3), (4), (5), and (6) and solving for the relevant total derivatives.

To see the intuition for Proposition 1, suppose the controlled rent, r_C, decreases while holding the uncontrolled rents, r_L and r_H fixed. Some higher-income households who previously avoided rent-controlled housing due to its lower quality may now find the reduced rent attractive enough to apply. As a result, the marginal applicants shift toward those who previously preferred high-quality housing, increasing the cutoff I_{HC}. Consequently, demand for high-quality housing in the uncontrolled sector declines.

Moreover, since rent-controlled units are allocated through a random rationing, the increase in applicants lowers the acceptance rate for all participants. This *crowding externality* affects those who, if rejected, would have rented uncontrolled low-quality housing. Consequently, even if the cutoff I_{HL} remains unchanged, more rejected applicants spill over into the uncontrolled low-quality segment, creating excess demand for these units.

The demand shift induces an increase in the supply of low-quality uncontrolled housing and a decrease in the supply of high-quality uncontrolled housing. This causes r_L and r_H to adjust in opposite directions as controlled rent decreases.

We define misallocation as the fraction of rent control applicants who end up in a different housing type than they would in an uncontrolled market, $\frac{\mu(F(I_{HC})-F(I_{HL}))}{F(I_{HC})} = \mu\left(1 - \frac{F(I_{HL})}{F(I_{HC})}\right)$.

Here, $F(I_{HC}) - F(I_{HL})$ represents the share of applicants who would have chosen high-quality housing but instead applied for rent-controlled units, while $\frac{\mu}{F(I_{HC})}$ is the acceptance rate. The following corollary characterizes how misallocation responds to changes in the controlled rent.

Corollary 1. *Housing misallocation decreases in the controlled rent.*

Proof. See Appendix A.1. □

A decrease in controlled rents raises uncontrolled low-quality rents while lowering high-quality rents. Consequently, more households apply for rent-controlled housing before considering high-quality housing, leading to an increase in I_{HC} in equilibrium. If rejected, the higher low-quality rents and lower high-quality rents make fewer household rents low-quality housing, I_{HL} decreases. Finally, misallocation increases as controlled rent decreases.

2.4 Effects of Rent Control Expansion

We now consider how rent control expansion—an increase in μ—affects rents in the two uncontrolled sectors. We hold h_H^0 fixed and allow h_L^0 to change, so the policy experiment re-designates some of the initial uncontrolled L-type stock as controlled housing. Formally, $\Delta h_L^0 = -\Delta(\mu n)$. We show that rent control expansion increases L-quality uncontrolled housing rents, even as it reduces H-quality uncontrolled housing rents.

Proposition 2. *The equilibrium rents of L-quality houses increase in the size of the rent controlled sector, while the equilibrium rents of H-quality houses decrease in the size of the rent controlled sector.*

Proof. See Appendix A.2. □

Proposition 2 is also formally shown by totally differentiating Equations (3), (4), (5), and (6) and solving for the relevant total derivatives.

To see the intuition for Proposition 2, consider an increase in μ by $\Delta\mu$, holding r_L and r_H fixed. This mechanically reduces the supply of uncontrolled L-quality housing by $n\Delta\mu$, while simultaneously decreasing the demand for both uncontrolled H-quality and L-quality housing by $\frac{\Delta\mu}{F(I_{HC})} n \left(F\left(I_{HC}\right) - F\left(I_{HL}\right)\right)$ and $\frac{\Delta\mu}{F(I_{HC})} n F\left(I_{HL}\right)$, respectively, due to the increased acceptance rate of rent controlled housing. In other words, while the reduction in supply occurs exclusively in the L-quality uncontrolled segment, due to random rationing, the reduction in demand is distributed across both L - and H-quality uncontrolled housing.

Since the newly occupied rent-controlled units were taken by households that would have otherwise resided in high-quality housing, the uncontrolled sector experiences excess demand for L-quality housing and excess supply in H-quality housing. In response, the housing supply system adjusts by decreasing the supply of H quality units while increasing the supply of L-quality units. This supply shift causes r_L and r_H to move in opposite directions as rent control expands.

In the linearized model, it is easy to characterize misallocation. We find that:

Corollary 2. *Housing misallocation increases in the size of the rent controlled sector.*

Proof. See Appendix A.2. □

The expansion of rent control (an increase in μ) lowers equilibrium H-quality rents, leading more households to rent H-quality housing directly instead of applying for controlled units, thus decreasing I_{HC}. Consequently, the acceptance rate $\frac{\mu}{F(I_{HC})}$ increases. At the same time, among those applying for controlled housing, more prefers H-quality over L-quality due to rent changes, reducing I_{HL}. Together, these effects increase the share of households renting H quality housing if rejected from controlled units -that is, $F\left(I_{HC}\right) - F\left(I_{HL}\right)$ increases, driven by the greater proportion of lower-income households or the concavity of the income CDF. As a result, misallocation $\frac{\mu(F(I_{HC})-F(I_{HL}))}{F(I_{HC})}$ increases.

3 Empirical Context

For the remainder of the paper, we assess the empirical relevance of our theory using data from Hong Kong. In this section, we provide relevant background on Hong Kong's public housing system and describe the data used for analysis.

3.1 Brief History of Hong Kong's Housing Policy

Hong Kong's public housing system was initially created in the 1950s to resettle a large refugee population that illegally resided in squatter areas. In the wake of the 1967 riots, Hong Kong initiated an aggressive urban development program to redress widespread discontent regarding housing. This program involved the development of rural areas into "New Towns." Hong Kong's Public Rental Housing (PRH) program, which provides subsidized rental housing, was greatly expanded. The Homeownership Scheme (HOS), which provides subsidized ownership housing, was also inaugurated.

However, the 1998 Asian Financial Crisis precipitated a deep recession and plummeting property prices in Hong Kong. In response, the Hong Kong Government repositioned its housing policy. Specifically, in 2002, it halted land auctions and suspended the Homeownership Scheme. Figure 3 Panel (a) shows that new public and private housing construction sharply declined in the early 2000s. For more than ten years, the construction of subsidized ownership units was almost zero. The average number of new housing units completed between 1997 and 2003 was 70,900, but declined to an average of 29,300 between 2004 and 2020. It was not until 2017 that new construction of subsidized ownership housing restarted.

Despite the slowdown in construction, Hong Kong's population grew at a similar rate between 1997 and 2019. Immigration from Mainland China was steady but restricted, in part to stabilize political sentiments after Hong Kong's sovereignty was transferred from Britain to China. Population growth was 0.6 percent per year on average during 1997-2019. It turned negative after 2019 in the wake of political unrest and the COVID-19 pandemic.

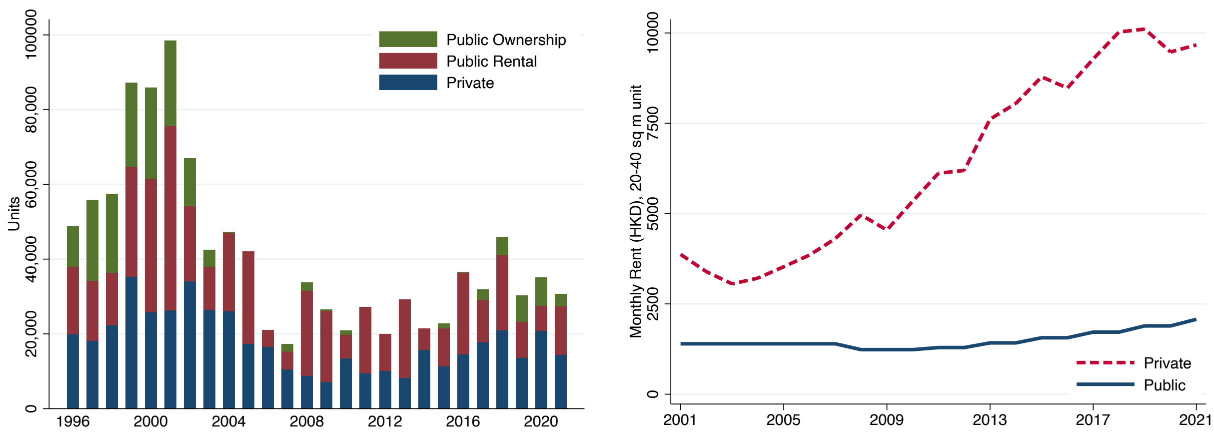
3.2 Data

We use data from five waves of the Hong Kong Census/By-Census spanning 2001 to 2021. Each wave collects a 5 percent sample of the total population every five years. The Hong Kong Census/By-Census data is the most appropriate resource for our research for two main reasons. First, it is the largest micro-level dataset available in Hong Kong, offering comprehensive demographic and income information at both the household and individual levels. Second, it includes detailed data on housing tenure types, which enables us to measure

Figure 3: Trends in construction and public-sector rents

(a) Public and Private Construction

(b) Public and Private Rents



Notes: Panel (a) plots the units of new construction, from 1996 to 2021. Units are divided into three categories: public ownership, public rental, and private. Panel (b) plots the trend of rent in PRH units and private housings. Private housings are constrained to those between 20 and 40 square meters and the private rental indices are weighted by the number of PRH units in each region in 2016 so as to be comparable with the PRH rental indices.

income and housing expenditures across various housing tenure groups and evaluate housing affordability

For our main analysis, we focus on data from 2006 to 2016, encompassing three waves in total. We exclude data prior to 2006 due to the impact of the Tenants Purchase Scheme, which allowed public housing tenants to buy their homes. We also exclude the 2021 data because household income was measured differently in that wave compared to previous ones. Additionally, we exclude all domestic helpers and workers in the household. In Hong Kong, foreign domestic helpers constitute a sizeable share of the total population. In the full sample set, domestic helpers and workers represent over 5.5% of the total observations. Their standardized contracts, fixed wages, and live-in requirements set them apart from the general labor force. Due to these structural differences, FDHs are typically excluded from inequality analyses to maintain accurate income distribution assessments.

Household income is defined as total monthly cash earnings from employment and other income sources of all household members. Housing expenditure is defined as monthly rent for renters and monthly mortgage payments for homeowners. Top coding affects very few observations of housing expenditure (0.03% of mortgage payments and 0.06% of rents), but is a more serious concern for household income (2.6% of the total observations). Our procedures for addressing top coding are described in Appendix B.1. Both incomes and expenditures are real values (take 1996 as the base year). We exclude observations with missing or zero household income after applying top-coding correction.

We equivalise both household income and housing expenditure by dividing the total amounts by the number of equivalent adults in the household and distributing equally among all household members.[1] More than 32% of individuals had zero equivalised housing expenditure; most of them private owners without a mortgage. Lastly, we define the housing expenditure share as the ratio of equivalised monthly housing expenditure to equivalised monthly household income. We exclude from the sample any individual who reports a housing expenditure share that is above two or below minus two.

3.3 Hong Kong's Housing Tenure Types

Due to Hong Kong's extensive public housing system, households can be categorized into four main tenure types: private renter, public renter, public owner, and private owner. The key features of each tenure type are discussed below. Tables A1 and A2 provide summary statistics.

Public Renters. Public renters live in subsidized and means-tested Public Rental Housing (PRH) units that are owned and operated by the Hong Kong government. These renters accounted for 33.4% of Hong Kong's population in 2006. PRH units are typically 300-400 square feet and assigned through a first-come, first-serve waiting list system. Applicants must satisfy both income and asset tests to receive a PRH unit. Residents also undergo regular means testing. PRH rents are determined by government policy, which mandates that renters exceeding certain income thresholds pay 1.5 times or double rent. Nominal rent increases are adjusted according to citywide income growth and capped at a maximum of 10% every two years. Figure 3 (b) shows that PRH rents remained stable even as private-sector rents

[1] We follow the "modified OECD" equivalence scale. The scale takes the first adult as having a weight of 1 point, each additional person over the age of 14 years is allocated 0.5 points, and each child under the age of 14 is allocated 0.3 points.

skyrocketed.

Public Owners. Public owners live in government-built ownership units without leasing or resale rights. They represented 18.0% of Hong Kong's population in 2006. A large majority of public owners live in Homeownership Scheme (HOS) units. The HOS units are usually 500-700 square feet, and their selling prices are often discounted by 35% to 50% relative to market valuations. These units are allocated to eligible buyers through a lottery mechanism. Another subset of public owners live in Tenant Purchase Scheme (TPS) units, which originated as PRH units offered for sale to sitting tenants at a deep discount between 1998 and 2006. Public owners cannot easily lease or resell their units without incurring a hefty premium.[2]

Private Owners. Private owners live in units that they can freely sell or lease to others as desired. As of 2006, private owners constituted only 35.6% of the population.

Private Renters. Private renters represent a small share of Hong Kong's population. These households lease their units from private-sector owners. As of 2006, private renters made up only 10.3% of the population.

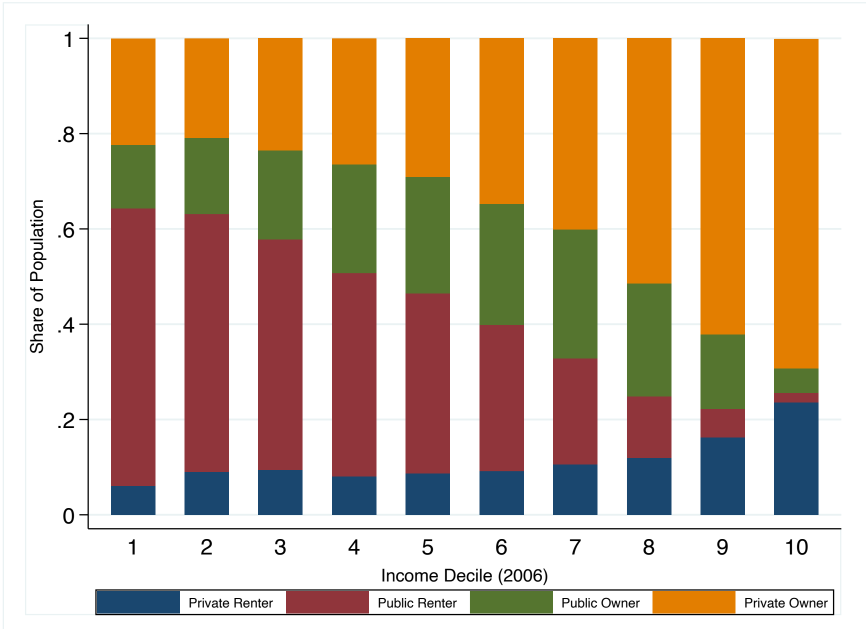
3.4 Distribution of Housing Tenure Types

As further background on Hong Kong's housing system, we provide a brief overview of the distribution of housing tenure types across income and age groups in 2006. The distribution of housing tenure varies markedly across incomes. Most low-income individuals live in public rental housing, and most high-income individuals own their homes. Additionally, lots of individuals aged 20-29 live with their parents and tend to move out when they get older.

Figure 4 illustrates the housing tenure distribution of Hong Kong's population by income decile in 2006. As indicated in the figure, public renters are primary households in the lower income deciles. Except for individuals in the highest income decile, public owners are relatively evenly distributed across the income distribution. The share of private owners rises

[2] All public owners are required to repay the initial discount at current market prices before gaining the right to rent out or resell their units. We classify TPS and HOS owners as private owners rather than public owners after they repay this discount. Since the discount repayment requirement is hefty, repayment is rare. As of 2023, less than 23% of HOS owners and 2% of TPS owners have not repaid the discount.

Figure 4: Housing tenure by income decile, 2006



Notes: Figure plots the distribution of housing tenure by income decile in 2006. Only individuals aged between 20 and 60 are included. For each income decile, the population is grouped into 4 tenure types: private renter, public renter, public owner, and private owner.

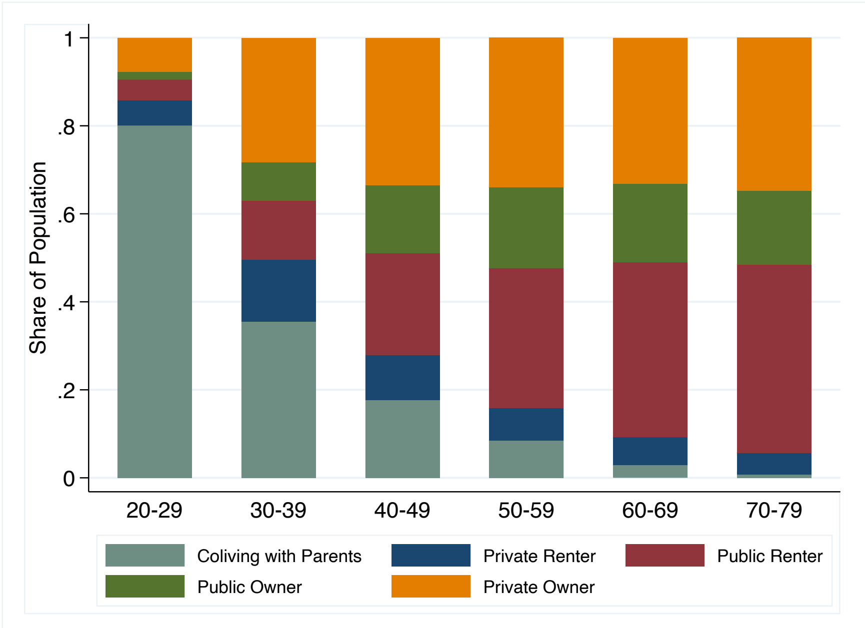
along with income decile. Private renters are distributed across various income groups, with a larger share in the high-income groups, which can be partially explained by the high-income and high-skilled expatriate workers residing in Hong Kong.

Figure 5 depicts the distribution of housing tenure types by age cohort in 2006. In the figure, individuals who live with their parents are shown as a separate category. Notably, in the 20–29 age group, 80.1% of individuals cohabit with their parents. This share drops to 17.7% for those aged 40-49.

The remaining individuals are sorted into the other four tenure types. The share of individuals who are private renters is highest among the 30-39 age group and then declines with age. The private renter share stands at 14.0% in the 30-39 age group but declines to 7.3% in the 50-59 age group. Some individuals move to public rental housing after getting old. This outcome likely reflects the priority given to elderly populations in the public housing allocation system.

The share of owners is quite stable after the age of 40. The private owner share is 33.1% for those aged 40–49 and also 33.1% for those aged 60-69. The public owner share is 15.5%

Figure 5: Distribution of Housing Tenure Types, by Age Group



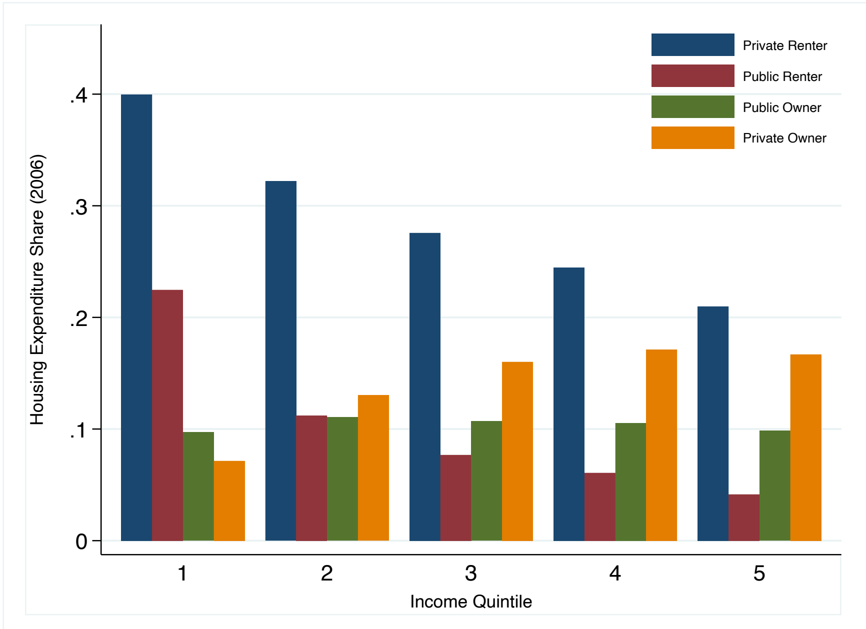
Notes: Figure plots the distribution of housing tenure types by age cohort in 2006. Only individuals aged between 20 and 79 are included, and they are grouped according to their age. Private renters, public renters, public owners, and private owners exclude members living with their parents.

for those aged 40–49 and 17.9% for those aged 60-69. These patterns suggest that many individuals decide to purchase an ownership unit before reaching age 40.

4 The Effects of Rent Regulation in Hong Kong

This section documents the impact of rent regulation in Hong Kong between 2006 and 2016. There are four main findings. First, rent regulation significantly distorted the distribution of housing expenditure shares across income segments. Second, public housing rent regulation shielded much of Hong Kong's population from rapidly rising private-sector housing expenditures. Third, public rental housing became increasingly misallocated towards middle-income households. Fourth, the unregulated prices, rents, and supply of low-quality housing disproportionately increased. Consistent with our theory, this descriptive evidence suggests that population growth exacerbated housing misallocation, contributing to a disproportionate growth in the demand for private rental units in Hong Kong's lower-quality segment.

Figure 6: Housing expenditure share by income quintile and housing tenure, 2006



Note: Figure plots the average housing expenditure share by income quintile and housing tenure in 2006. Only individuals aged between 20 and 60 are included. The housing expenditure share is calculated as the ratio of equivalised monthly housing expenditure over equivalised monthly income. Income quintiles are defined by the equivalised monthly household income. Individuals are divided into four housing tenure types: private renters, public renters, public owners, and private owners.

4.1 Inequality in Housing Expenditure Shares

First, we show that rent regulation was a key determinant of cross-sectional differences in housing expenditure shares in Hong Kong's population. Using disaggregated population data, we find that the housing expenditure shares of public renters are much lower than private renters. Moreover, the cross-sectional relationship between housing expenditure share and income in Hong Kong is U-shaped, due to a large segment of middle-income individuals living in subsidized housing. These findings suggest that the public housing sector significantly distorts housing expenditures in Hong Kong.

Figure 6 plots housing expenditure shares—defined as the share of income spent on housing expenditure—by household income and housing tenure type in 2006. We find that individuals living in public housing had a much lower expenditure share than those living in private housing across all income quintiles. For instance, in the bottom income quintile, private renters averagely spend 40.0% of their income on rents, while public renters averagely spend 22.5% of their income on rent. Meanwhile, in the bottom income quintile, public

Table 1: Regression of housing expenditure share on tenure types and income

Dependent Variable	Housing Expenditure Share					
Survey Year	2006			2016		
	(1)	(2)	(3)	(4)	(5)	(6)
Public Renter	-0.152***	-0.125***	-0.112***	-0.263***	-0.245***	-0.237***
	(0.00132)	(0.00131)	(0.00138)	(0.000955)	(0.000973)	(0.00104)
Public Owner	-0.176***	-0.144***	-0.152***	-0.308***	-0.288***	-0.291***
	(0.00138)	(0.00138)	(0.00143)	(0.00106)	(0.00109)	(0.00114)
Private Owner	-0.121***	-0.101***	-0.110***	-0.237***	-0.225***	-0.231***
	(0.00123)	(0.00122)	(0.00122)	(0.000909)	(0.000920)	(0.000929)
Log real HH income	-0.0236***	-0.0269***	-0.0352***	-0.0336***	-0.0346***	-0.0407***
	(0.000478)	(0.000481)	(0.000493)	(0.000396)	(0.000397)	(0.000406)
Household Demographics		X	X		X	X
Unit Characteristics			X			X
Observations	197103	197103	197059	197548	197548	197548
Adjusted R^2	0.085	0.136	0.159	0.353	0.371	0.384

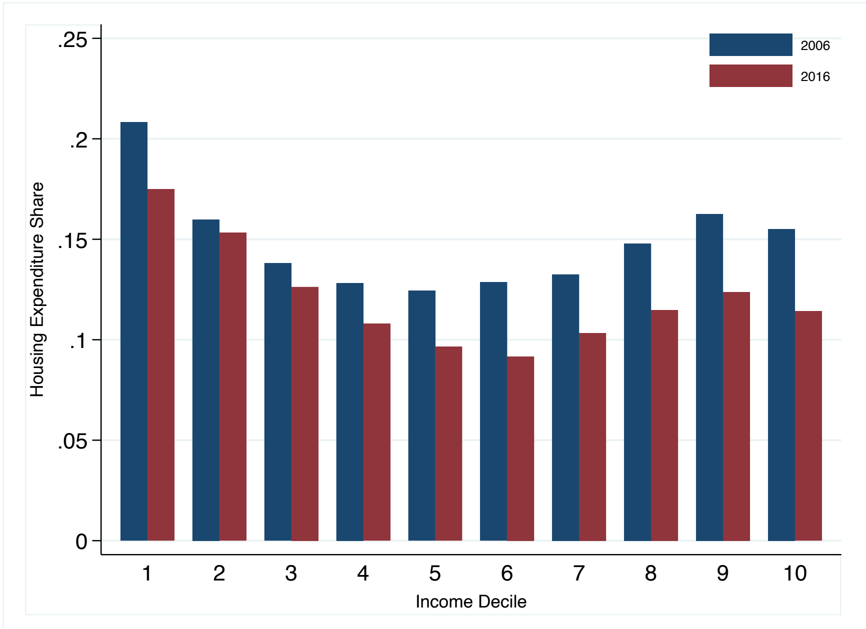
Note: The average housing expenditure shares for private renters in 2006 and 2016 are 27.1% and 33.5%, respectively. Only individuals aged between 20 and 60 are included. *Household demographics* include household size, number of children aged below 15, number of elders aged over 60, dummy of immigrant, and age. *Unit characteristics* include dummies of 18 districts and number of rooms. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

owners spend 9.7% of their income on housing, and private owners spend 7.1% of their income on housing. A similar pattern is found across the income distribution.

Within the rental sector, housing expenditure shares decline as income increases. Among private renters, the housing expenditure share for individuals in the bottom income quintile is 40.0%, but it falls to 21.0% for individuals in the top income quintile. Intriguingly, housing expenditure shares also fall with income among public renters, from 22.5% in the bottom quintile to 4.1% in the top quintile. Within the ownership sector, housing expenditure shares increase with income. Among private owners, the housing expenditure share for households in the bottom income quintile is 7.1%, and it increases to 16.7% for households in the top income quintile. Housing expenditure shares slightly increase across the income distribution for public owners, at around 11%. Table 1 shows that a large rent differential between private and public renters remains after controlling for household and unit characteristics.

Figure 7 plots the average housing expenditure share in each income decile in 2006 and in 2016. The plot reveals an unusual U-shape relationship: in Hong Kong, the housing expenditure share of middle-income individuals is much lower than that of both high- and

Figure 7: Housing expenditure share by income decile, 2006 and 2016



Note: Figure plots the average housing expenditure share by income decile in 2006 and 2016. Only individuals aged between 20 and 60 are included. The housing expenditure share is calculated as the ratio of equivalised monthly housing expenditure over equivalised monthly income. Income deciles are also defined by the equivalised monthly household income in each year.

low-income individuals. In 2006, individuals in the lowest-income decile spent, on average, 20.8% of their income on housing, which is 8 percentage points more than the average for middle-income individuals. High-income individuals also had an average housing expenditure share of around 16 percent. This unusual U-shaped pattern—where both low- and high-income individuals spend a higher share of their income on housing compared to middle-income individuals—was still present in 2016. This pattern is unusual because, in most other countries, the share of income spent on housing is monotonically decreasing in household income.[3]

The U-shape relationship between housing expenditure share and income is explained by two facts: (1) higher-income populations are much more likely to reside in the private sector than middle-income populations, and (2) private-sector housing expenditure shares are much higher throughout the income distribution. As shown in Figure 4, in the middle

[3]For example, Figure 7 Panel (a) in Dustmann, Fitzenberger and Zimmermann (2022) shows that the housing expenditure share decreases from 39% in the lowest income quintile to 14% in the highest income quintile in Germany in 2013. Moreover, Figure 1 in Larrimore, Schuetz et al. (2017) shows that the portion of income spending on rent decreased from 56% in the lowest income quintile to 10% in the highest income quintile in United States in 2015. Those two figures claim that the standard pattern of the housing expenditure share is monotonically decreasing across income. It reflects the speciality of the Hong Kong housing market.

quintile, 62% of individuals live in public housing. However, in the top quintile, only 7% live in public housing. Moreover, as previously shown in Figure 6, housing expenditures are much higher for private-sector residents.

4.2 Trends in Housing Expenditure Shares

Having shown that rent regulation significantly suppressed the housing expenditure shares of public renters, we next show that rent regulation also helped to insulate the vast majority of Hong Kong residents from rising private-sector rental housing expenditures. Using decomposition methods, we find that only private renters—which account for a small fraction of the local population—saw large increases in housing expenditures between 2006 and 2016. Meanwhile, housing expenditure shares fell in almost all other housing tenure groups.

Figure 8 plots the change in housing expenditure share by income quintile and housing tenure between 2006 and 2016. We find that individuals living in private rental housing were the only group to see an increase in housing expenditure shares across income levels. The increase in housing expenditure shares among private renters is very large. In the four lowest income quintiles, housing expenditure shares increased by 5-8 percentage points.

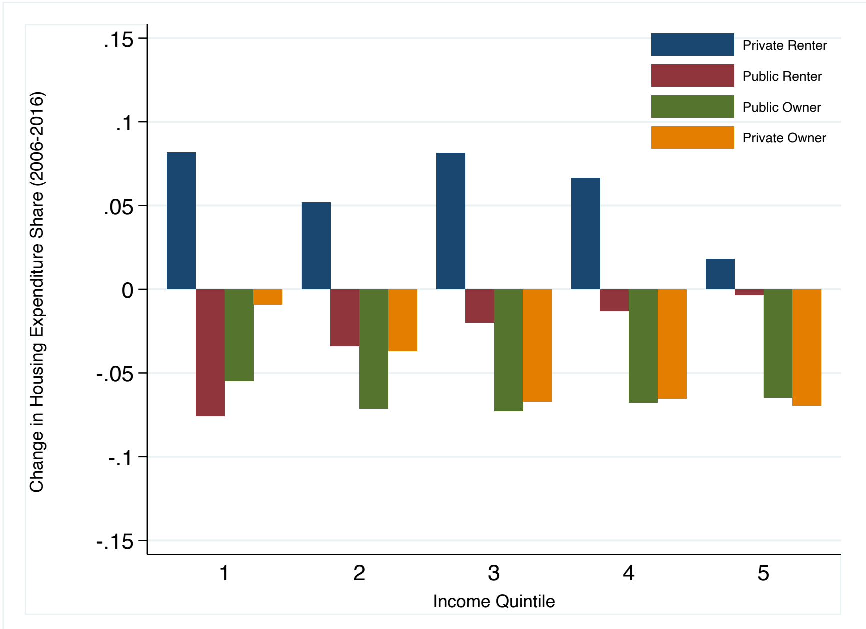
By contrast, for public renters, the housing expenditure share fell significantly. In the lowest income quintile, the housing expenditure share of public renters decreased by 7.6 percentage points, from 22.5% to 14.9%. In higher income deciles, the decline was smaller, but this is largely attributable to the fact that housing expenditure shares for public renters are already exceedingly low, as previously shown in Figure 6.

For both private and public owners, housing expenditure shares significantly fell between 2006 and 2016, by around 7 percentage points, respectively. As shown previously in Figure 4, private and public owners make up roughly 75% of individuals in the top income quintile. Therefore, the decline in housing expenditures among owners explains 127.5% of the 4.0 p.p. decline in housing expenditure shares for individuals in the top income quintile.[4]

These changes are not explained by changes in household or unit composition. Table 1

[4]In the top income quintile, the overall decline of housing expenditure share from 2006 to 2016 is 4.0 percentage points. 75% of individuals are owners in the top quintile, whose housing expenditure share decreases 6.8 percentage points. Thus, 127.5% ($\frac{0.75\times 6.8}{4.0}$) of the overall decline in housing expenditure share in the top income quintile is due to the decline in private and public owners.

Figure 8: Change in housing expenditure share by income quintile and housing tenure



Note: Figure plots the change in housing expenditure share by household income quintile and housing tenure type. Only individuals aged between 20 and 60 are included. The housing expenditure share is calculated as the ratio of equivalised monthly housing expenditure over equivalised monthly income. The within-group change is calculated as the difference in share between 2006 and 2016 for each housing tenure type and income quintile.

Columns (3) and (6) show that even after controlling household and unit characteristics, the private-public rent differential grew from 11.2 log points to 23.7 log points between 2006 and 2016. Housing tenure also became a much larger driver of housing expenditure inequality in 2016 than in 2006. Comparing Column (1) from 2006 with Column (4) from 2016, the adjusted R^2 increases from 0.085 to 0.353. This difference suggests that housing tenure types account for a much larger proportion of the variation in the housing expenditure share in 2016 compared to 2006.

Despite the dramatic increase in private-sector housing expenditures, Figure 7 shows that the average housing expenditure share decreased in all income deciles from 2006 to 2016. The decline in low-income individuals is somewhat less than that in middle- and high-income individuals. For instance, the expenditure share in the second income deciles fell by roughly 0.6 percentage points. By contrast, the expenditure share in the top two income deciles fell by roughly 4 percentage points.

Appendix Table A3 decomposes the change in housing expenditure shares between

2006 and 2016 within each income quintile into components explained and unexplained by observable household characteristics using the Blinder-Oaxaca method (Blinder 1973; Oaxaca 1973). Specifically, we estimate separate regressions of the housing expenditure share for each income quintile on four sets of observable characteristics: household demographics, unit characteristics, tenure type, and household income.[5] The results confirm that the changes in housing expenditure shares are not explained by observable characteristics.

4.3 Trends in Housing Tenure Distribution

Having established that rent regulation helped to moderate the impact of rising housing costs for public renters, we now examine how the patterns of household sorting into public housing changed between 2006 and 2016. Specifically, we decompose changes in the population's housing tenure distribution across income and age groups. We find that the proportion of the population living as public renters increases in the middle of the income distribution but falls in the bottom of the income distribution. Meanwhile, younger cohorts and low-income immigrants shift more toward private rental units.

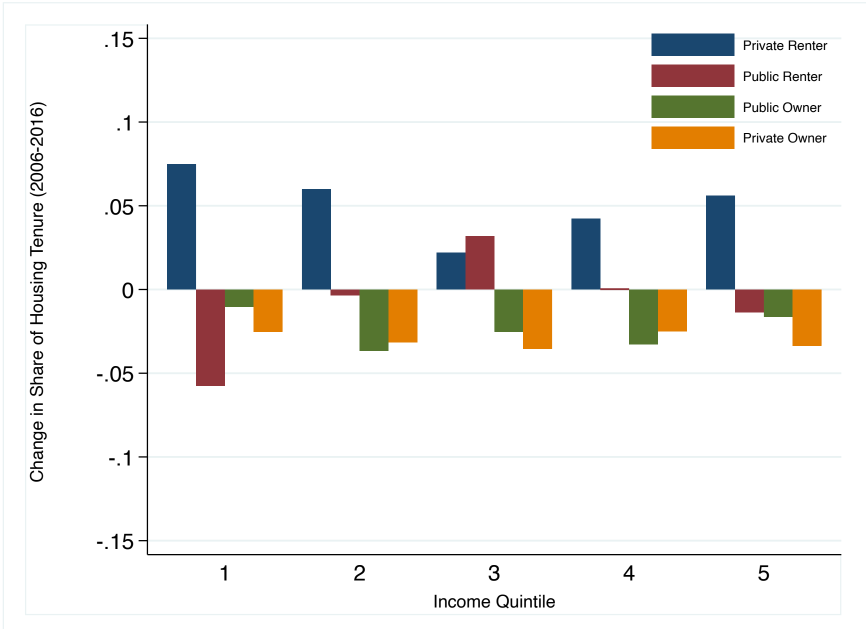
Figure 9 shows that the share of the population living as private renters rose in all income groups. In the bottom two income quintiles, the private renter share rose by 6-8 p.p. In the top quintile, the increase is also large, at about 7 p.p. However, the increase in private renter share in the middle quintile is smaller, at about 2 p.p.

Meanwhile, the share of population living as public renters in the middle-income quintile increased by about 3 percentage points. At the same time, the share of public renters in the lowest income quintile fell by more than 5.5 percentage points.The increase in middle-income population and the decline in low-income population living as public renters suggest that public rental housing—which was intended for low-income households—was progressively misallocated to the relatively well-off between 2006 and 2016.

In addition, the share of population living as private and public owners fell across the income distribution. In all income quintiles, the drop in the private owner share ranged from 3-4 percentage points. In the middle three quintiles, the public owner share declined by around 3 percentage points. These declines indicate greater difficulty in moving upwards on

[5]A detailed explanation of the Blinder-Oaxaca decomposition is provided in Appendix C.

Figure 9: Change in housing tenure type share by income quintile



Note: Figure plots the change in housing tenure type share by household income quintile and housing tenure type. Only individuals aged between 20 and 60 are included. The housing tenure type share is defined as the ratio of the population living in each type over the total population by income quintile. Individuals are divided into four housing tenure types: private renters, public renters, public owners, and private owners. The within-group change is calculated as the difference in share between 2006 and 2016 by each income quintile.

the housing ladder.

Table 2 decomposes the change in public renter share among renters within income quintiles between 2006 and 2016 into within-subgroup and between-subgroup components, according to the following equation:

$$\Delta s^q_{PRH} = \sum_g s^g_{q,16} \Delta s^{q,g}_{PRH} + \sum_g s^q_{PRH,06} \Delta s^g_q,$$

where q indexes income quintile, g indexes demographic subgroup, Δ denotes the change between 2006 and 2016, s^q_{PRH} denotes the public renter share among renters, and s^g_q denotes the share of subgroup g among renters within income quintile q.

The decline in public renter share is found to be especially large for immigrants in lower-income quintiles, as well as for non-elderly households in lower-income quintiles and for households without children. Among the lowest-income immigrants, public renter share fell by nearly 29 p.p. Renters living without elderly underwent a much larger decline in public renter

Table 2: Decomposition of changes in public renter share by income quintiles, 2006-2016

	Quintile of equivalised household income				
	Q1	Q2	Q3	Q4	Q5
A. Overall					
Initial public renter share	88.15%	84.03%	79.51%	61.38%	17.76%
Change in public renter share	-11.32%	-8.96%	-2.19%	-7.07%	-8.25%
B. Immigrant status					
Initial immigrant share	22.91%	14.33%	7.57%	6.17%	16.17%
Change in immigrant share	-1.45%	1.80%	1.76%	1.77%	3.65%
Change in public renter share					
Within immigrants	-29.10%	-26.40%	-8.14%	-8.44%	-3.76%
Within natives	-6.57%	-5.40%	-1.19%	-6.33%	-8.68%
Within-group component	-11.73%	-8.41%	-1.72%	-6.46%	-7.88%
C. Living with elderly					
Initial share of renters living with elderly	28.37%	26.73%	25.21%	20.25%	9.54%
Change in share of renters living with elderly	0.50%	2.72%	3.92%	2.81%	-0.48%
Change in public renter share					
Within renters living with elderly	-4.93%	-3.61%	-3.04%	-8.89%	-16.26%
Within renters living without elderly	-13.97%	-11.61%	-2.71%	-7.54%	-7.29%
Within-group component	-11.41%	-9.47%	-2.79%	-7.81%	-8.14%
D. Living with child					
Initial share of renters living with child	51.17%	35.92%	23.48%	22.06%	31.26%
Change in share of renters living with child	-10.32%	-7.57%	-4.46%	-1.39%	6.91%
Change in public renter share					
Within renters living with child	-18.71%	-12.58%	-10.05%	-15.13%	-6.79%
Within renters living without child	-6.38%	-8.01%	-0.90%	-5.42%	-7.92%
Within-group component	-12.69%	-9.65%	-3.05%	-7.56%	-7.57%

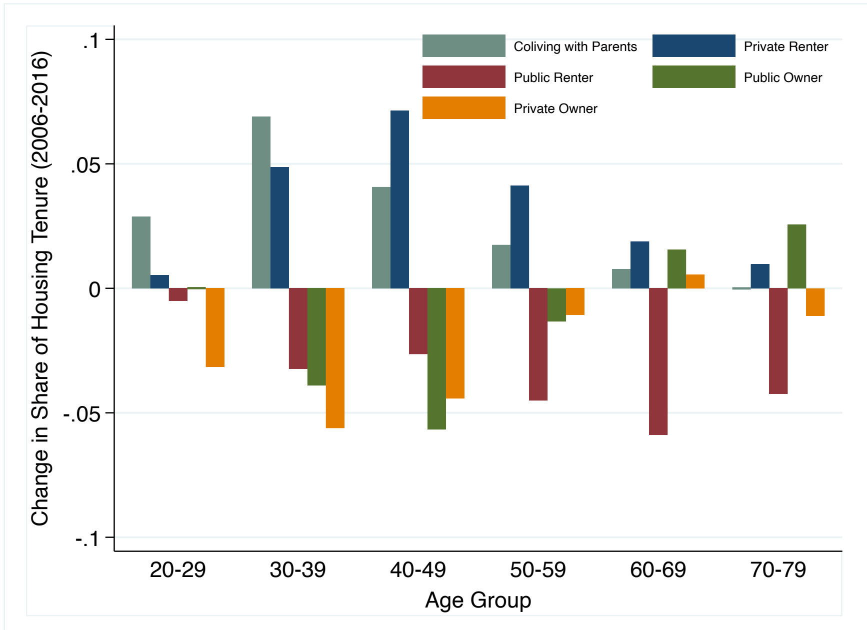
Notes: This table presents the decomposition of changes in public renter share among renters by immigrant status and family structure across quintiles of equivalised household income (Q1 to Q5) from 2006 to 2016. *Immigrant* is defined as individuals whose place of born is not Hong Kong and living in Hong Kong for less or equal to 10 years. *Living with elderly* is defined as individuals living with at least one member aged over 60. *Living with child* is defined as individuals living with at least one member aged less than or equal to 15. The *within-group component* captures changes in public housing share occurring within each subpopulation, holding their relative sizes constant. Negative (positive) values imply a decrease (increase) in the respective shares.

share compared to living with elderly. Renters living with children also saw a much larger decline in public renter shares compared to those without children. The disproportionate impacts on non-elderly households without children are likely due to the fact that elderly populations are prioritized by the public rental queue.[6]

Because of these large declines within subgroups, the overall changes in public renter shares stem from shifts in public renter shares within subgroups rather than from changes in

[6] Appendix Figure A2 further decompose individuals by living with elderly or children after controlling their immigrant status.

Figure 10: Change in Housing Tenures by Age Group



Notes: Figure plots the change in housing tenure between 2006 and 2016. Only individuals aged between 20 and 79 are included, and they are grouped according to their age. Private renters, public renters, public owners, and private owners exclude members living with their parents. The within-group change is calculated as the difference in share between 2006 and 2016 by each age group.

population composition. Appendix Table A4 uses the Blinder-Oaxaca method to decompose the changes in public renter shares within income deciles into components that are either explained or not explained by changes in observable characteristics. The results confirm that the decline in the share of public renters remains significant even after holding constant observable characteristics.

Figure 10 shows the changes in housing tenure distribution by age group from 2006 to 2016. The figure reveals a marked rise in the number of individuals living with their parents across all age groups. The increase is most pronounced for individuals aged 30–39, whose share living with parents increased from 34.3% to 41.3%. For those aged 50–59, the share expanded from 8.3% to 10.1%. Similarly, the share of private renters grew most among 40–49 year olds, from 10.0% to 17.1%. Meanwhile, the share living as public renters fell similar across age groups. For individuals aged 30–39, it fell from 13.0% to 9.8%. For individuals aged 50–59, the share fell from 31.2% to 26.7%. These changes suggest that younger adults were increasingly exposed to rising private rental costs.

Appendix Table A5 decomposes the change in public renter share among renters by educational background. It is found that the share of public renters among renters decreased in all age groups from 2006 to 2016. The steepest decline is observed among individuals aged 30–49. An especially large decline is observed among younger adults who did not complete upper secondary education, suggesting that low-income young adults were increasingly exposed to rising private rental costs.[7]

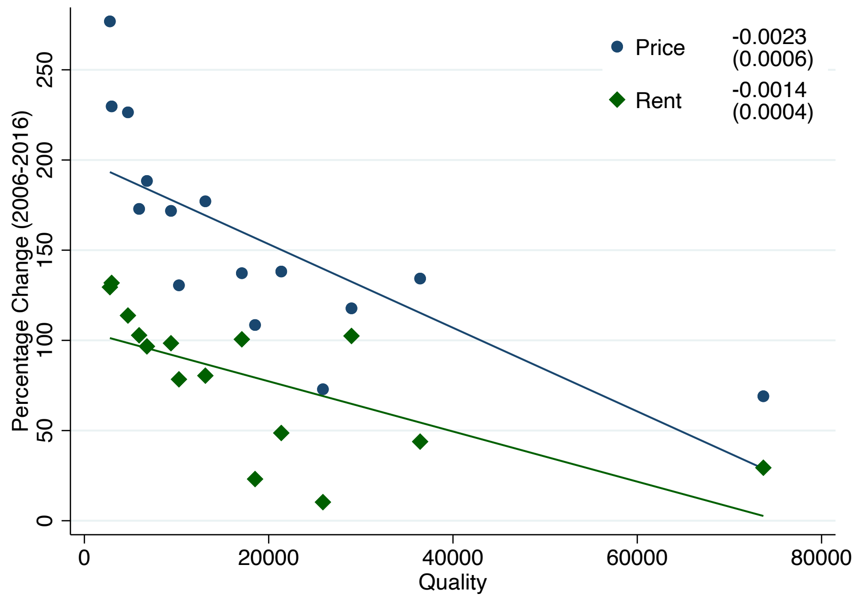
4.4 Trends in Unregulated Rents and Housing Supply

The previous subsection documented a large increase in private renter population, especially among lower-income population. This increase in the private-sector population may alter prices and supply in the private market. We now examine changes in private-sector price, rents, and supply by quality segment. Our main finding is that prices, rents, and supply of low-quality uncontrolled homes increased disproportionately. Although this change cannot be causally attributable to reduced, it is suggestive that that housing demand shifted to lower quality segments.

Figure 11 shows that prices and rents increased disproportionately in the lower-quality segments. We plot changes in prices and rents between 2006 and 2016 against housing quality. We measure the change in price and rent using indices for each district and unit size class published by the Hong Kong Rating and Valuation Department (RVD). For each quality segment, we compute a quality index using average private-sector rents by district and unit size in the 2016 Hong Kong Population Census. Prices and rents have increased disproportionately in the lower-quality segments. On Hong Kong Island, for instance, prices and rents for the smallest units (Class A) rose by 226.4% and 113.8%, respectively, compared to 69.0% and 29.4% for the largest units (Class E) between 2006 and 2016. The downward-

[7] According to the Hong Kong Census and Statistics Department, there are four main categories of educational attainment: primary or below, lower secondary, upper secondary, and post-secondary. *Upper secondary* includes secondary 4–7 of the old academic structure (1985–2011), secondary 4–6 of the new academic structure (2012 onwards), Project Yi Jin/Yi Jin diploma, diploma of applied education, and craft level. *Post-secondary* includes diploma, certificate, higher certificate, higher diploma, professional diploma, associate degree, pre-associate degree, endorsement certificate, associateship, first degree, taught postgraduate, and research postgraduate courses.

Figure 11: Change in prices and rents by quality segment

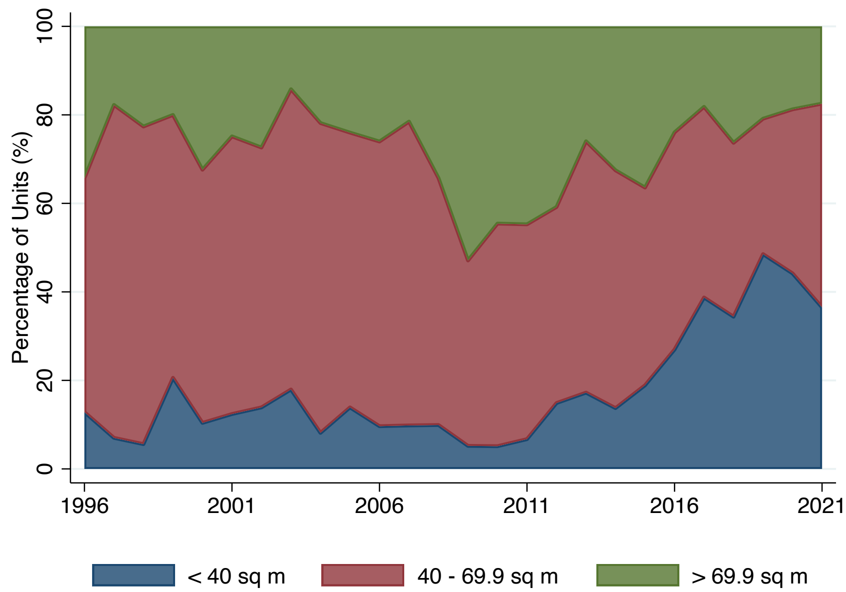


Notes: Figure plots percentage change in prices and rents between 2006 and 2016 against unit quality. Price and rental indices are observed from RVD data. Each dot represents one quality segment, which is comprised of one region and one unit class. Set of regions contains Hong Kong Island, Kowloon and Territories. There are 5 area classes in total, from A to E. The horizontal axis shows the unit quality. Regression coefficients and standard error (in parenthesis) are included in the legend.

sloping regression line further highlights these disparities, underscoring a clear pattern of uneven growth. Appendix Figure A3 shows that the changes in the price-to-rent ratio are not correlated with housing quality.

Figure 12 shows that the new construction of smaller units has also grown considerably, especially relative to larger units. New construction of Class B units (40 m^2–69.9 m^2) dominated the private market until 2018, when it was surpassed by Class A units (<40 m^2). Previously, Class A units accounted for only 5% to 20% of new private housing supply, but this figure surged to a peak of 48.5% in 2019. Meanwhile, the share of Class B units fell from a peak of 75.2% in 1997 to 30.6% in 2019.

Figure 12: Trend in new construction



Notes: Figure plots the proportion of newly completed units in the private market, categorized by area classes from 1996 to 2021.

5 Conclusion

This paper provides new theory and evidence on the equilibrium effects of rent regulation. The novelty here is to relax the assumption in existing rent control models that uncontrolled houses are perfectly divisible by building on a growing literature on housing assignment models. We also incorporate flexible supply substitution patterns. We show that rent control not only misallocates housing, but may also have opposite effects on the rents of low-quality and high-quality units in the uncontrolled sector. We characterize conditions under which rent regulation exacerbates rather than alleviates the harmful consequences of inadequate affordable housing for a subpopulation of low-income households.

We offer evidence consistent with the model by documenting the evolution of housing expenditure inequality and housing misallocation in Hong Kong. We use various decomposition methods to understand disaggregated population, rent, price, and construction data. We find that public housing insulated a large fraction of households from rising private-sector housing costs between 2006-2016. However, public housing became increasingly misallocated.

The demand for smaller private-sector units disproportionately increased. The result was a dramatic increase in the rents of small units, as well as a disproportionate burden borne by young renters and migrants. These findings confirm the empirical relevance of housing assignment models for understanding the equilibrium effects of housing policy on different segments of the population within urban metro areas.

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A Proofs

A.1 Proof of proposition 1

For ease of notation, define the partial derivatives as follows:

$$l_\mu \equiv \frac{\partial h_L}{\partial \mu}, \quad l_l \equiv \frac{\partial h_L}{\partial r_L}, \quad l_h \equiv \frac{\partial h_L}{\partial r_H},$$
$$h_l \equiv \frac{\partial h_H}{\partial r_L}, \quad h_h \equiv \frac{\partial h_H}{\partial r_H}.$$

Given the conditions $\theta_H > \theta_L > 0$ and $\phi'', p' > 0$, Equations (7) and (8) imply that:

$$\begin{aligned}
l_l &= \frac{1}{\phi''} > 0, \\
l_h &= -\frac{\theta_L}{\theta_H}\frac{1}{\phi''} < 0, \\
h_l &= -\frac{\theta_L}{\theta_H}\frac{1}{\phi''} < 0, \\
h_h &= \left(\frac{\theta_L}{\theta_H}\right)^2 \frac{1}{\phi''} + \left(\frac{1}{\theta_H}\right)^2 \frac{1}{p'} > 0.
\end{aligned}$$

It then follows that:

$$\begin{aligned}
l_l h_h - h_l l_h &> 0, \\
l_l + h_l &= \left(1 - \frac{\theta_L}{\theta_H}\right)\frac{1}{\phi''} > 0, \\
l_h - h_l &= 0 \\
l_l + l_h + h_l + h_h &= \left(1 - \frac{\theta_L}{\theta_H}\right)^2 \frac{1}{\phi''} + \left(\frac{1}{\theta_H}\right)^2 \frac{1}{p'} > 0.
\end{aligned}$$

Given $\Delta h_L^0 = -\Delta(\mu n)$, we have $l_\mu = -n$.

For exposition, we now define new notations:

$$U_{CL} \equiv u_c(I_{HC} - r_C, L), \qquad U_{CH} \equiv u_c(I_{HC} - r_H, H),$$
$$U_L \equiv u_c(I_{HL} - r_L, L), \qquad U_H \equiv u_c(I_{HL} - r_H, H),$$
$$F_C \equiv F(I_{HC}), \qquad F_L \equiv F(I_{HL}).$$

Note that:

- $F_C - \mu > 0$
- $I_{HC} > I_{HL}$, $F_C > F_L$, and $F'_C, F'_L > 0$
- $U_{CH} > U_{CL} > 0$ and $U_H > U_L > 0$, since $u_c, u_{cq} > 0$, $u_{cc} < 0$.
- $(l_l + h_l)U_H + (l_h + h_h)U_L > (l_l + l_h + h_l + h_h)U_L > 0$.

We first totally differentiate Equations (3), (4), (5), and (6) with respect to r_C. This yields the following system of equations:

$$\begin{cases} (U_{CH} - U_{CL}) \dfrac{dI_{HC}}{dr_C} - U_{CH} \dfrac{dr_H}{dr_C} = -U_{CL} \\ (U_H - U_L) \dfrac{dI_{HL}}{dr_C} - U_H \dfrac{dr_H}{dr_C} + U_L \dfrac{dr_L}{dr_C} = 0 \\ nF_C (F_C - \mu) F'_L \dfrac{dI_{HL}}{dr_C} + \mu n F_L F'_C \dfrac{dI_{HC}}{dr_C} - F_C^2 l_h \dfrac{dr_H}{dr_C} - F_C^2 l_l \dfrac{dr_L}{dr_C} = 0 \\ (l_h + h_h) \dfrac{dr_H}{dr_C} + (l_l + h_l) \dfrac{dr_L}{dr_C} = 0 \end{cases} \tag{9}$$

Solving this system, we get:

$$\begin{bmatrix} \frac{dI_{HL}}{dr_C} & \frac{dI_{HC}}{dr_C} & \frac{dr_H}{dr_C} & \frac{dr_L}{dr_C} \end{bmatrix} = \begin{bmatrix} \frac{K_1}{K} & \frac{K_2}{K} & \frac{K_3}{K} & \frac{K_4}{K} \end{bmatrix},$$

where

$$K = (U_{CH} - U_{CL})\, F_C \Bigg((U_H - U_L) \Big(F_C \underbrace{(h_l l_h - h_h l_l)}_{<0} - \underbrace{n\,(F_C - \mu)\, F_L'}_{>0} \underbrace{((l_l + h_l) U_H + (l_h + h_h) U_L)}_{>0} \Big) \Bigg)$$
$$- \underbrace{n\mu (l_l + h_l) U_{CH} F_L (U_H - U_L) F_C'}_{>0} < 0$$

and

$$K_1 = -n\mu U_{CL} F_L F_C' \left(U_H \left(l_l + h_l \right) + U_L \left(l_h + h_h \right) \right) < 0$$
$$K_2 = F_C U_{CL} \Bigg(F_C \left(U_H - U_L \right) \underbrace{(l_l h_h - l_h h_l)}_{>0} + n F_L' \left(F_C - \mu \right) \underbrace{\Big(\left(l_l + h_l \right) U_H + \left(l_h + h_h \right) U_L \Big)}_{>0} \Bigg) > 0$$
$$K_3 = -n\mu U_{CL} F_L F_C' \left(U_H - U_L \right) \left(l_l + h_l \right) < 0$$
$$K_4 = n\mu U_{CL} F_L F_C' \left(U_H - U_L \right) \left(l_h + h_h \right) > 0$$

It follows that $\frac{dI_{HL}}{dr_C} > 0$, $\frac{dI_{HC}}{dr_C} < 0$, $\frac{dr_H}{dr_C} > 0$. By Assumption 2, we have:

$$l_h + h_h = \left(\frac{1}{\theta_H} \right)^2 \frac{1}{p'} + \frac{\theta_L}{\theta_H} \left(\frac{\theta_L}{\theta_H} - 1 \right) \frac{1}{\phi''} > 0$$

$\frac{dr_L}{dr_C} = \frac{K_4}{K} < 0$ if and only if $K_4 < 0$, which holds if and only if $l_h + h_h > 0$, which in turn holds if and only if $\phi'' > \theta_L(\theta_H - \theta_L)p'$.

Then we prove Corollary 1. Misallocation is defined as $\frac{\mu(F(I_{HC}) - F(I_{HL}))}{F(I_{HC})} = \mu \left(1 - \frac{F(I_{HL})}{F(I_{HC})} \right)$. Because I_{HC} decreases and I_{HL} increases in r_C, misallocation decreases in r_C.

A.2 Proof of proposition 2

We next totally differentiate Equations (3), (4), (5), and (6) with respect to μ. This yields the following system of equations:

$$\begin{cases} \left(U_{CH}-U_{CL}\right)\dfrac{dI_{HC}}{d\mu}-U_{CH}\dfrac{dr_H}{d\mu}=0, \\ \left(U_H-U_L\right)\dfrac{dI_{HL}}{d\mu}-U_H\dfrac{dr_H}{d\mu}+U_L\dfrac{dr_L}{d\mu}=0, \\ nF_C\left(F_C-\mu\right)F_L'\dfrac{dI_{HL}}{d\mu}+\mu nF_LF_C'\dfrac{dI_{HC}}{d\mu}-F_C^2l_h\dfrac{dr_H}{d\mu}-F_C^2l_l\dfrac{dr_L}{d\mu}=nF_C\left(F_L-F_C\right) \\ \left(l_h+h_h\right)\dfrac{dr_H}{d\mu}+\left(l_l+h_l\right)\dfrac{dr_L}{d\mu}=0. \end{cases} \tag{10}$$

Solving this system, we obtain:

$$\begin{bmatrix} \frac{dI_{HL}}{d\mu} & \frac{dI_{HC}}{d\mu} & \frac{dr_H}{d\mu} & \frac{dr_L}{d\mu} \end{bmatrix} = \begin{bmatrix} \frac{K_5}{K} & \frac{K_6}{K} & \frac{K_7}{K} & \frac{K_8}{K} \end{bmatrix},$$

where $K<0$ has been defined as above, and

$$K_5 = nF_C\underbrace{\left(F_C-F_L\right)}_{>0}\underbrace{\left(U_{CH}-U_{CL}\right)}_{>0}\underbrace{\Big(\left(l_l+l_h\right)U_H+\left(l_h+h_h\right)U_L\Big)}_{>0}>0$$

$$K_6 = nF_C\left(F_C-F_L\right)\left(l_l+h_l\right)U_{CH}\left(U_H-U_L\right)>0$$

$$K_7 = nF_C\left(F_C-F_L\right)\left(l_l+h_l\right)\left(U_{CH}-U_{CL}\right)\left(U_H-U_L\right)>0$$

$$K_8 = -\underbrace{nF_C\left(F_C-F_L\right)\left(U_{CH}-U_{CL}\right)\left(U_H-U_L\right)}_{>0}\left(h_h+l_h\right)$$

It follows that $\frac{dI_{HL}}{d\mu}$, $\frac{dI_{HC}}{d\mu}$ and $\frac{dr_H}{d\mu}<0$. Moreover, $\frac{dr_L}{d\mu}=\frac{K_8}{K}>0$ if and only if $K_8<0$, which holds if and only if $l_h+h_h>0$, which in turn holds if and only if $\phi''>\theta_L(\theta_H-\theta_L)p'$.

Then we prove Corollary 2. Since I_{HC}, hence $F(I_{HC})$ decreases in μ, if $F(I_{HC})-F(I_{HL})$ increases in μ, then misallocation $\frac{\mu(F(I_{HC})-F(I_{HL}))}{F(I_{HC})}$ increases in μ.

I_{HL} also decreases in μ. With Cobb-Douglas utility, $I_{HC}-I_{HL}=\sigma\left(r_L-r_C\right)$ increases in μ. Hence, if $F(\cdot)$ is (weakly) concave, then $F(I_{HC})-F(I_{HL})$ increases in μ.

B Data Construction

B.1 Top-coding Correction

Top-coding is a critical concern in survey data because it affects both the calculation of average income and the analysis of inequality trends at the upper end of the income distribution. In Hong Kong Census data from 2006 to 2016, the public-use censoring point for monthly household income is $150,000, for mortgage and rental payments are $99,998. Following previous literature (see, e.g. Heathcote, Perri and Violante (2010), Domeij and Ljungqvist (2019), Heathcote et al. (2023)), we implement a regression approach to address this issue. Specifically, we assume that the upper tail of the income distribution follows a Pareto distribution. We fit a Pareto distribution to the uncensored upper tail of the observed data and use this fitted distribution to estimate mean values for top-coded observations. This approach dynamically adjusts to variations in top-coding thresholds over different years and mitigates internal censoring, ensuring continuity and consistency in the data.

B.2 Construction of Rental Indices for PRH Units and Comparable Private Homes

In Figure 3(b), we construct a rental measure for PRH units and private housing which is comparable to PRH units. To ensure comparability, only housing between 20 and 40 square meters included and the private rental indices are weighted by the region and district distribution of PRH units in 2016. It is noted that 67.2% of the PRH stock in 2016 have the size between 20 and 40 square meters.

First, we construct the rental indices for PRH units in which we normalize the PRH rent in 2016 to one, and then adjust yearly with the percentage increase in PRH rents announced by the Housing Authority (HA). HA will review and adjust the rent every two years, and the rate of rent increase is capped at 10%. Then, the public rental indices are multiplied by the average rent of 20 to 40 square meter PRH units measured in the 2016 Population Census to obtain the sequence of public rent shown in Figure 3(b).

Next, we show how we compute the figure for comparable private units. The first step is to calculate a weighted average private rent. Using the number of PRH units each district in 2016 as weights, we take the weighted average of the private rent of untis between 20 and 40 square meters in each district. Then, using the rental indices data from the Rating and Valuation Department (RVD), we compute the average private rental indices for class A units (units with area less then 40 quare meters), weighted by the number of PRH units in each region (Hong Kong Island, Kowloon and New Territories). Rental indices in 2016 are normalized to 1. For RVD data, we only have access to rental indices by region but not by district. In Hong Kong, there is 3 regions and 18 districts, which can be respectively categorized into the 3 regions. Eventually, we acquire the time series sequence of comparable private units by multiplying the normalized rental indices with the weighted average private rent in 2016 computed in the first step.

B.3 Quality Measurement of Private Rental Housing Across Years

In this part of the appendix, we discuss our measurement of quality of private rental housing, as presented in Figure 11 and Appendix Figure A4.

In order to measure quality of housing, we set 2006 as our base year such that rental values of private housing across different census years should be normalized to that in 2006 and therefore become comparable across years.

We divide housing into 15 quality segments, each of which consists of one region out of three and one size category out of five, which is consistent with the way Rating and Valuation Department (RVD) measures price and rental indices. The three regions are Hong Kong Island, Kowloon and New Territories respectively; and the five size categories are A ($< 40m^2$), B ($40m^2 - 69.9m^2$), C ($70m^2 - 99.9m^2$), D ($100m^2 - 159.9m^2$) and E ($> 160m^2$). It should be noted that classes (i.e. size categorizes) are only available in the population census in and after 2016. We, for example, would like to measure the housing quality in 2016 and our quality measurement is as follows.

First, we calculate the mean rent of private housing under each quality segment in 2016 Population Census and then discount the present rental value by the growth in rental indices in that segment during the period concerned to estimate the relevant rental value in the base year. Then, we set up a linear regression model with estimated percentage change in rental values against the logarithmic value of the estimated base year rental values. This step is to construct a simple model to measure change in rental values over the period concerned. Finally, through the regression model, we can estimate the base year rental value by observing their rents in 2016 from the census data. The estimated base year rental value is the quality of the particular unit, and is comparable across different census years.

C Derivation of Blinder-Oaxaca Decomposition

This section explains Blinder-Oaxaca decomposition in detail and introduces further empirical results from the decomposition. Studies by Oaxaca (1973) and Blinder (1973) analyze the difference in wages between different genders and races in United States. They provided a new econometric strategy called Oaxaca-Blinder Decomposition, which separately quantifies the explained and unexplained factor of the difference. For the housing expenditure share in each year, we consider the following function:

$$Y_i^g = m^g\left(X_i, \epsilon_i\right) = \beta_0^g + \beta_1^g X_{1i} + \cdots + \beta_K^g X_{Ki} + \epsilon_i, \quad \text{for } g = 2006, 2016 \tag{11}$$

where Y^{2006} are housing expenditure shares according to the structure of households in 2006, Y^{2016} are housing expenditure shares according to the structure of households in 2016. X_i is a data matrix including all observed and unobserved characteristics.

In our analysis, there are four groups of observed characteristics, $K = 4$. This function includes two key points: (1) we assume that the effect of observed and unobserved characteristics are additively separable in $m()$; (2) given X and G, the error term ϵ_i is purely random. Therefore, we can decompose the difference of housing expenditure share between two years

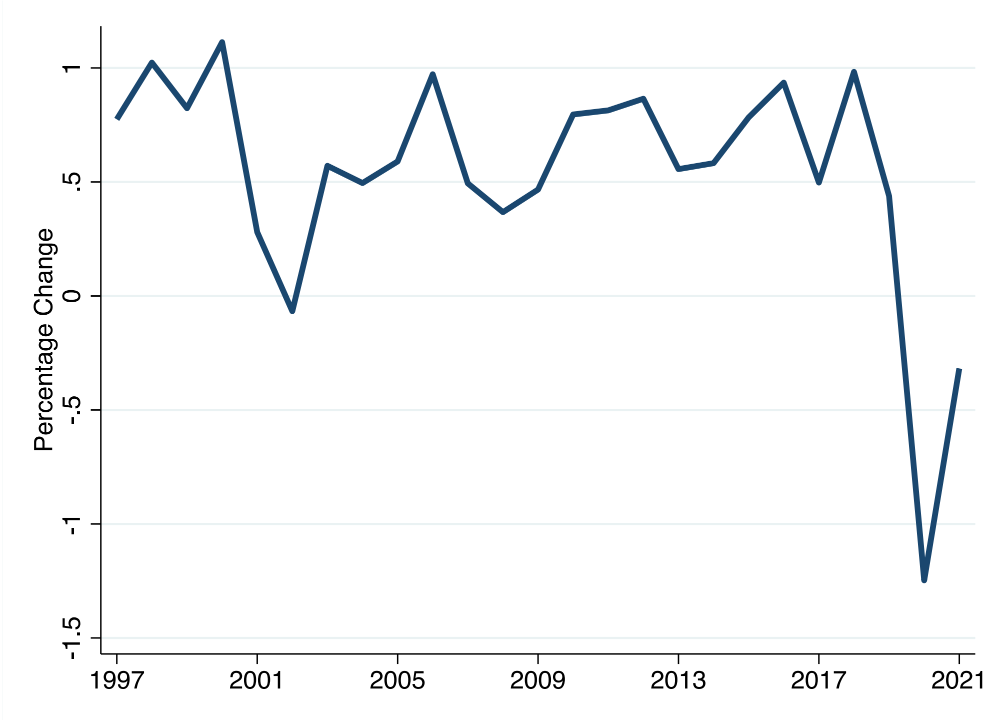
(Δ_μ) as following

$$
\begin{aligned}
\Delta^\mu &= \mathrm{E}(Y \mid G=2016) - \mathrm{E}(Y \mid G=2006) \\
&= \mathrm{E}\left(X\beta_{16} + \epsilon \mid G=2016\right) - \mathrm{E}\left(X\beta_{06} + \epsilon \mid G=2006\right) \\
&= \left(\mathrm{E}\left(X\beta_{16} \mid G=2016\right) + \mathrm{E}(\epsilon \mid G=2016)\right) - \left(\mathrm{E}\left(X\beta_{06} \mid G=2006\right) - \mathrm{E}(\epsilon \mid G=2006)\right) \\
&= \mathrm{E}\left(X\beta_{16} \mid G=2016\right) - \mathrm{E}\left(X\beta_{06} \mid G=2006\right) \\
&= \mathrm{E}(X \mid G=2016)\beta_{16} - \mathrm{E}(X \mid G=2006)\beta_{06} \\
&= \mathrm{E}(X \mid G=2016)\beta_{16} - \mathrm{E}(X \mid G=2006)\beta_{16} + \mathrm{E}(X \mid G=2006)\beta_{16} - \mathrm{E}(X \mid G=2006)\beta_{06} \\
&= \left(\mathrm{E}(X \mid G=2016) - \mathrm{E}(X \mid G=2006)\right)\beta_{16} + \mathrm{E}(X \mid G=2006)\left(\beta_{16} - \beta_{06}\right) \\
&= \Delta_E + \Delta_{UE}
\end{aligned}
\tag{12}
$$

where Δ_E denotes the explained portion and Δ_{UE} denotes the unexplained portion. Control variables X include four key factors: household demographics, unit characteristics, housing tenure types, and household income, as defined in the notes to Table A3. Following the last step in Equation 12, we can use the sample OLS regression to estimate the explained and unexplained components.

D Additional Tables and Figures

Figure A1: Population Growth, 1997 - 2021



Notes: Figure plots the percentage change in Hong Kong population. Population figures are recorded and reported by the Hong Kong Census and Statistics Department.

Table A1: Summary Statistics (Renters)

	2001	2006	2011	2016	2021
A. Public Renter					
Population	2,115,980	2,114,400	2,135,580	2,066,620	2,099,440
As share of a total population	34.0 %	33.4 %	31.9 %	31.0 %	31.0 %
Number of HHs	630,740	696,940	747,860	751,100	749,260
Share below PRH income limit	55 %	55 %	62 %	61 %	62 %
Real HH income	15,918	14,433	14,091	17,290	18,352
	(11,765)	(11,801)	(10,947)	(13,416)	(14,102)
Real Housing expenditure	1,426	1,627	1,297	1,410	1,792
	(665)	(752)	(649)	(739)	(868)
Average HH size	3.4	3.0	2.9	2.8	2.8
Share with members aged ≤ 15	36.0 %	32.1 %	26.2 %	20.5 %	19.7 %
Share with members aged > 60	45.2 %	42.1 %	46.0 %	51.6 %	61.0 %
Share moved in last 5 years	29.2 %	29.7 %	20.0 %	15.5 %	15.1 %
B. Private Renter					
Population	694,080	649,880	792,940	995,800	1,009,040
As share of a total population	11.2 %	10.3 %	11.8 %	14.9 %	14.9 %
Number of HHs	299,420	283,920	332,180	405,460	414,900
Share below PRH income limit	28 %	22 %	23 %	31 %	31 %
Real HH income	32,164	37,254	45,043	40,909	42,411
	(47,387)	(53,488)	(58,483)	(50,313)	(42,944)
Real Housing expenditure	8,142	8,515	9,894	10,701	10,824
	(16,588)	(17,560)	(14,888)	(12,349)	(12,235)
Average HH size	2.3	2.3	2.4	2.5	2.6
Share with members aged ≤ 15	32.1 %	31.4 %	33.9 %	34.3 %	33.0 %
Share with members aged > 60	16.9 %	14.4 %	15.6 %	18.6 %	25.0 %
Share moved in last 5 years	74.9 %	68.6 %	71.9 %	64.4 %	59.7 %

Notes: Table summarizes the summary statistics of private and public renters respectively, using the 5% sample of the Hong Kong Population Census from 2001 to 2021. *Share below PRH income limit* is denoted as the ratio of individuals with average household income lower than PRH income limit. *Household income* is defined as the sum of earnings in cash from all employment and other cash income. *Housing expenditure* for renters includes basic rent, while housing expenditure for owner-occupiers includes mortgage payment. Both incomes and expenditures are real values (take 1996 as the base year). The Standard deviations are reported in parentheses.

Table A2: Summary Statistics (Owners)

	2001	2006	2011	2016	2021
A. Public Owner					
Population	1,086,900	1,142,340	1,152,040	1,072,600	1,085,840
As share of a total population	17.5 %	18.0 %	17.2 %	16.1 %	16.0 %
Number of HHs	311,660	344,820	368,480	365,760	381,980
Share below PRH income limit	26 %	26 %	29 %	38 %	43 %
Real HH income	25,687	24,374	25,112	26,553	26,298
	(19,064)	(20,291)	(20,148)	(22,073)	(21,851)
Real Housing expenditure	3,722	2,554	1,404	944	1,060
	(4,252)	(3,718)	(2,645)	(2,555)	(2,857)
Average HH size	3.5	3.3	3.1	2.9	2.9
Share with members aged ≤ 15	44.4 %	33.6 %	24.3 %	20.0 %	17.5 %
Share with members aged > 60	30.0 %	32.9 %	40.6 %	51.2 %	65.7 %
Share moved in last 5 years	35.6 %	9.1 %	9.0 %	5.2 %	9.6 %
Share with zero mortgage	41.0 %	55.7 %	70.8 %	81.5 %	83.1 %
B. Private Owner					
Population	2,117,000	2,259,660	2,400,020	2,257,600	2,336,100
As share of a total population	34.1 %	35.6 %	35.8 %	33.9 %	34.4 %
Number of HHs	711,720	797,040	858,840	822,480	860,900
Share below PRH income limit	22 %	21 %	23 %	26 %	29 %
Real HH income	39,439	38,308	42,353	45,994	45,193
	(48,719)	(47,469)	(51,525)	(51,812)	(43,647)
Real Housing expenditure	6,455	5,717	4,129	3,949	4,566
	(10,945)	(10,524)	(8,218)	(7,460)	(8,511)
Average HH size	3.0	2.8	2.8	2.7	2.9
Share with members aged ≤ 15	35.7 %	31.2 %	28.0 %	26.2 %	25.0 %
Share with members aged > 60	31.0 %	28.8 %	34.8 %	40.4 %	50.3 %
Share moved in last 5 years	37.9 %	30.6 %	26.7 %	19.3 %	21.2 %
Share with zero mortgage	54.8 %	54.9 %	59.9 %	63.8 %	63.9 %

Notes: Table summarizes the summary statistics of private and public owners respectively, using the 5% sample of the Hong Kong Population Census from 2001 to 2021. *Share below PRH income limit* is denoted as the ratio of individuals with average household income lower than PRH income limit. *Household income* is defined as the sum of earnings in cash from all employment and other cash income. *Housing expenditure* for renters includes basic rent, while housing expenditure for owner-occupiers includes mortgage payment. Both incomes and expenditures are real values (take 1996 as the base year). The Standard deviations are reported in parentheses.

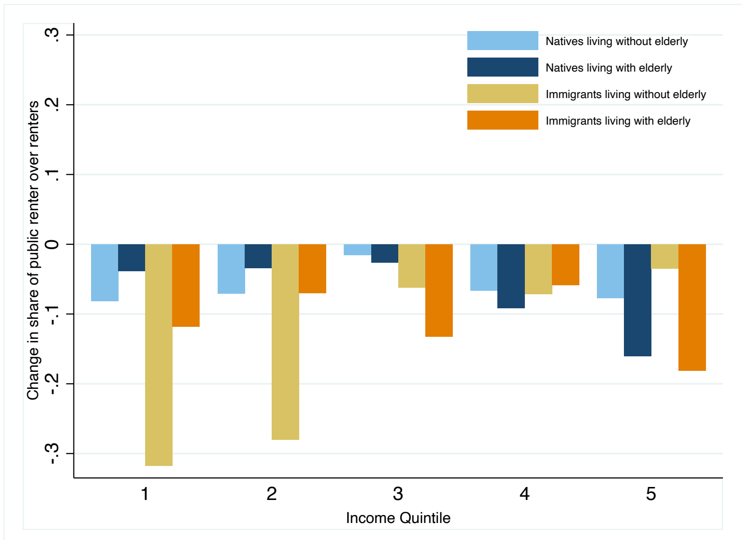
Table A3: Blinder-Oaxaca Decomposition of change in housing expenditure share, 2006 to 2016

	Quintile of equivalised household income				
	Q1	Q2	Q3	Q4	Q5
A. Aggregate decomposition					
2006	0.1869***	0.1353***	0.1281***	0.1433***	0.1626***
	(0.0009)	(0.0008)	(0.0008)	(0.0009)	(0.0008)
2016	0.1687***	0.1202***	0.0958***	0.1118***	0.1244***
	(0.0010)	(0.0009)	(0.0008)	(0.0008)	(0.0007)
Change	-0.0183***	-0.0151***	-0.0323***	-0.0315***	-0.0382***
	(0.0014)	(0.0012)	(0.0011)	(0.0012)	(0.0011)
Explained Δ_E	0.0038***	0.0068***	0.0003	-0.0014	-0.0118***
	(0.0010)	(0.0017)	(0.0018)	(0.0014)	(0.0006)
Unexplained Δ_{UE}	-0.0220***	-0.0219***	-0.0326***	-0.0301***	-0.0264***
	(0.0013)	(0.0018)	(0.0020)	(0.0015)	(0.0010)
B: Explained Δ_E					
Household Demographics	0.0023***	0.0008**	-0.0011***	-0.0000	-0.0042***
	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0004)
Unit Characteristics	0.0004**	0.0000	0.0005**	0.0007***	0.0020***
	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0003)
Tenure Type	0.0186***	0.0128***	0.0018***	0.0036***	0.0040***
	(0.0008)	(0.0005)	(0.0004)	(0.0003)	(0.0002)
Income	-0.0176***	-0.0068***	-0.0010	-0.0056***	-0.0135***
	(0.0006)	(0.0015)	(0.0018)	(0.0012)	(0.0004)
Observations	79353	78499	78929	78920	78906

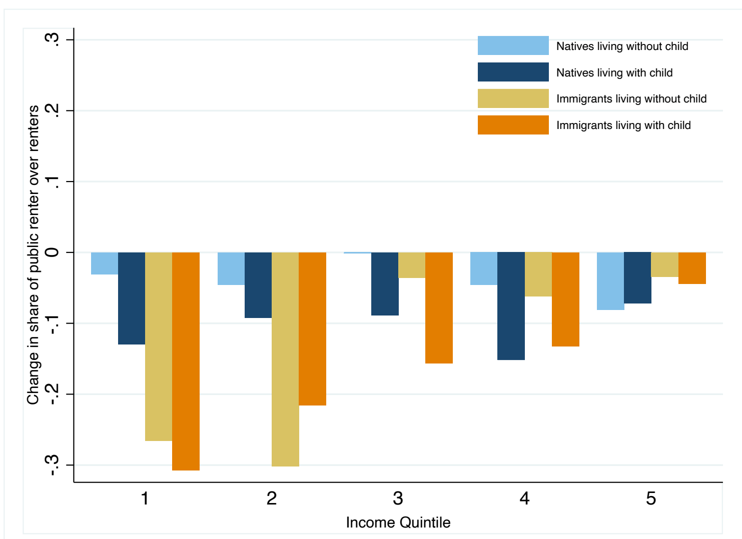
Note: The table shows a Blinder-Oaxaca decomposition of changes in housing expenditure share separately for each equivalised household income. *Household demographics* includes household size, number of elders over 60, number of children less than 15, age, and a dummy for immigrants. *Unit characteristics* include dummies of 18 districts and number of rooms. *Tenure type* includes dummies for being a public renter, public owner, private renter, or private owner. *Income* includes a log equivalised HH income. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure A2: Changes in public renter share by income quintile, 2006-2016

(a) Immigrant status × Living with elderly



(b) Immigrant status × Living with child



Notes: Figure plots the public renter share over renters in 2006 and 2016 by income quintiles, residential statuses, and family structures. Only individuals aged between 20 and 60 are included. The share is defined as the ratio of the population living in public rental housing over total renters by income quintile. *Immigrant* is defined as individuals whose place of birth is not Hong Kong and living in Hong Kong for less or equal to 10 years. *Living with elderly* is defined as individuals living with at least one member aged over 60. *Living with child* is defined as individuals living with at least one member aged less than or equal to 15

Table A4: Blinder-Oaxaca Decomposition of change in public renter share, 2006-2016

	Quintile of equivalised household income				
	Q1	Q2	Q3	Q4	Q5
A. Aggregate decomposition					
2006	0.8815***	0.8403***	0.7951***	0.6138***	0.1776***
	(0.0020)	(0.0025)	(0.0031)	(0.0045)	(0.0039)
2016	0.7684***	0.7507***	0.7732***	0.5431***	0.0951***
	(0.0026)	(0.0028)	(0.0030)	(0.0043)	(0.0028)
Change	-0.1132***	-0.0896***	-0.0219***	-0.0707***	-0.0825***
	(0.0033)	(0.0038)	(0.0043)	(0.0062)	(0.0048)
Explained Δ_E	-0.0005	0.0027***	0.0057***	0.0051**	-0.0125***
	(0.0006)	(0.0007)	(0.0011)	(0.0021)	(0.0017)
Unexplained Δ_{UE}	-0.1127***	-0.0924***	-0.0276***	-0.0758***	-0.0700***
	(0.0033)	(0.0037)	(0.0041)	(0.0058)	(0.0045)
B. Explained Δ_E					
Living with child	-0.0019***	0.0015***	0.0031***	0.0030***	-0.0067***
	(0.0004)	(0.0004)	(0.0005)	(0.0011)	(0.0008)
Living with elderly	0.0005	0.0028***	0.0056***	0.0068***	-0.0014
	(0.0004)	(0.0004)	(0.0007)	(0.0013)	(0.0012)
Immigrant	0.0009***	-0.0015***	-0.0030***	-0.0047***	-0.0045***
	(0.0002)	(0.0003)	(0.0006)	(0.0009)	(0.0007)
Observations	51788	45375	36711	25095	20670

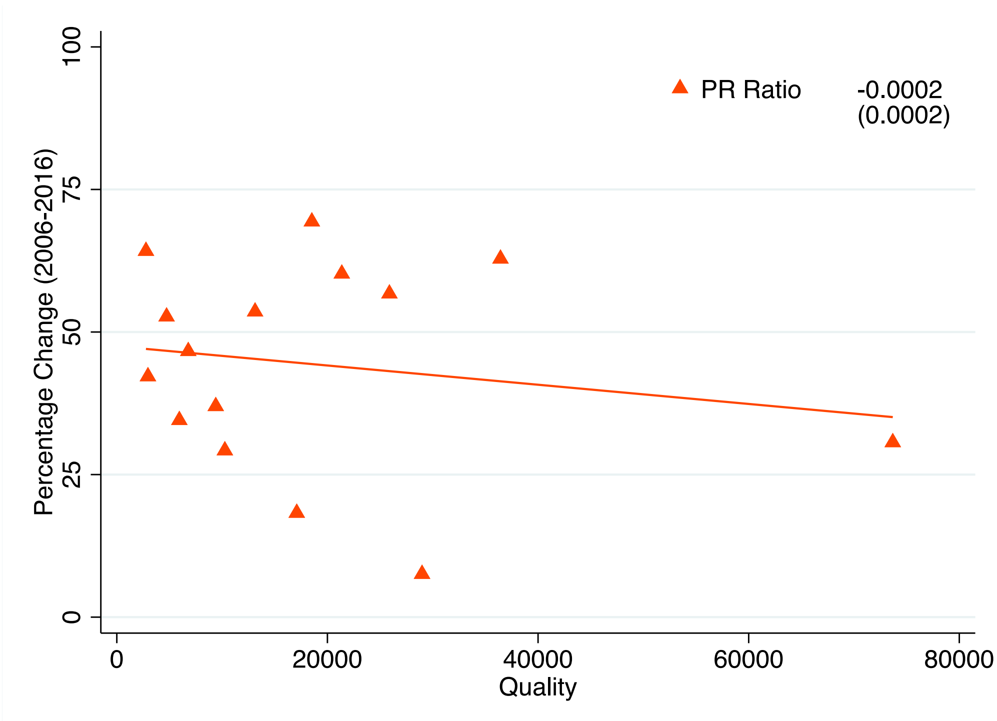
Note: The table shows a Blinder-Oaxaca decomposition of changes in public renter share over renters separately for each equivalised net household income. *Immigrant* is defined as individuals whose place of born is not Hong Kong and living in Hong Kong for less or equal to 10 years. *Living with elderly* is defined as individuals living with at least one member aged over 60. *Living with child* is defined as individuals living with at least one member aged less than or equal to 15. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A5: Decomposition of changes in public renter share by age groups, 2006-2016

	Age Group					
	20-29	30-39	40-49	50-59	60-69	70-79
A. Overall						
Initial public renter share	44.69%	48.89%	69.24%	81.26%	86.38%	89.66%
Change in public renter share	-5.01%	-14.29%	-15.61%	-11.04%	-5.87%	-2.99%
B. Educational background						
Initial H-educ share	73.44%	59.53%	39.64%	21.84%	16.54%	7.22%
Change in H-educ share	13.93%	15.64%	16.44%	18.81%	5.03%	9.57%
Change in public renter share						
Within H-educ	1.06%	-4.40%	-9.51%	-5.20%	-10.58%	-2.77%
Within L-educ	-25.90%	-17.88%	-9.54%	-4.99%	-3.18%	-1.22%
Within-group component	-6.10%	-9.85%	-9.53%	-5.03%	-4.40%	-1.33%

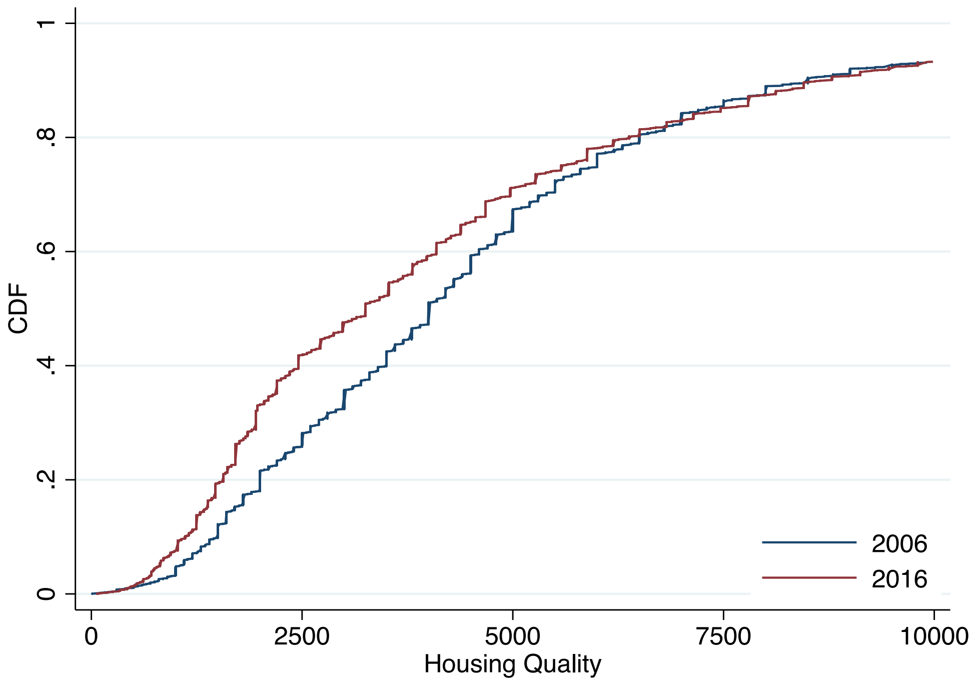
Notes: This table presents the decomposition of changes in public renter share by educational backgrounds across five age groups from 2006 to 2016. *H-educ* is defined as individuals who completed upper secondary courses or above. The *within-group component* captures changes in public housing share occurring within each subpopulation, holding their relative sizes constant. Negative (positive) values imply a decrease (increase) in the respective shares.

Figure A3: Change in Price-to-rent Ratio by quality segment, 2006-2016



Notes: Figure plots percentage change in price-to-rent ratio between 2006 and 2016 against unit quality. Price and rental indices are observed from RVD data. P/R ratio is computed by the ratio of price index to annual rental index. Each dot represents one quality segment, which is comprised of one region and one unit class. Set of regions contains Hong Kong Island, Kowloon and Territories. There are 5 area classes in total, from A to E. The horizontal axis shows the unit quality. Regression coefficient and standard error (in parenthesis) are included in the legend.

Figure A4: Distribution of Housing Quality, 2006 and 2016



Notes: Figure plots the cumulative distribution function (CDF) of private rental housing quality in 2006 and 2016. Growth in rental indices amongst different classes between 2006 and 2016 are observed in RVD data, and household's rent in 2016 census is used to estimate their rent in 2006 by the relevant growth in the unit class. Note that class of units is only observable after 2016 Census. An OLS regression is then used to estimate the linear rent growth equation by considering growth in rent (the dependent variable) and estimated rental value in 2006 (the independent variable). Observed in 2016 Census, household rents are turned back to into 2006's estimated value through the OLS estimated linear rent growth equation. Only households with real household income between $5000 and $45,000 are included.